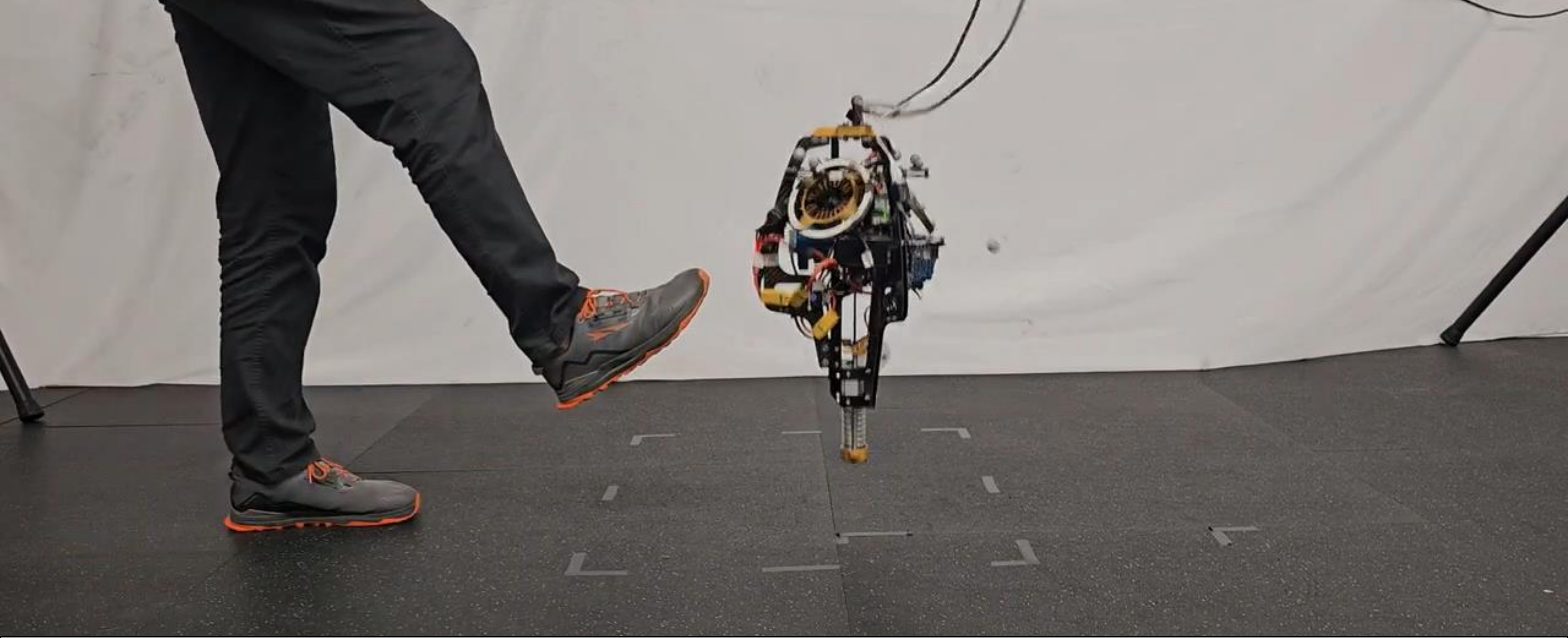
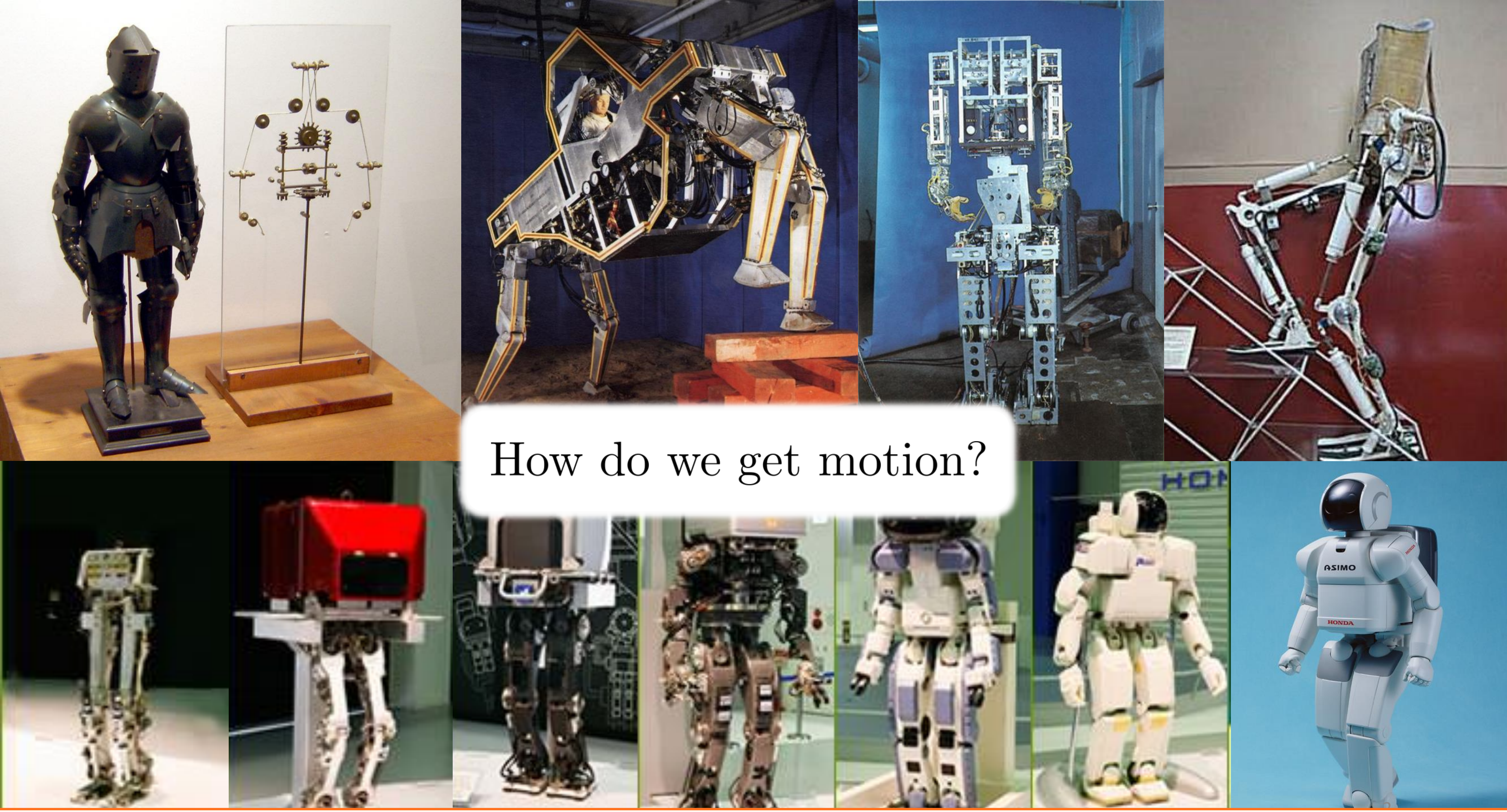


Hierarchical Robotic Control:
Constructive Theory and Application to Legged Systems

Noel Csomay-Shanklin
11/1/24

AMBER LAB

Caltech



How do we get motion?

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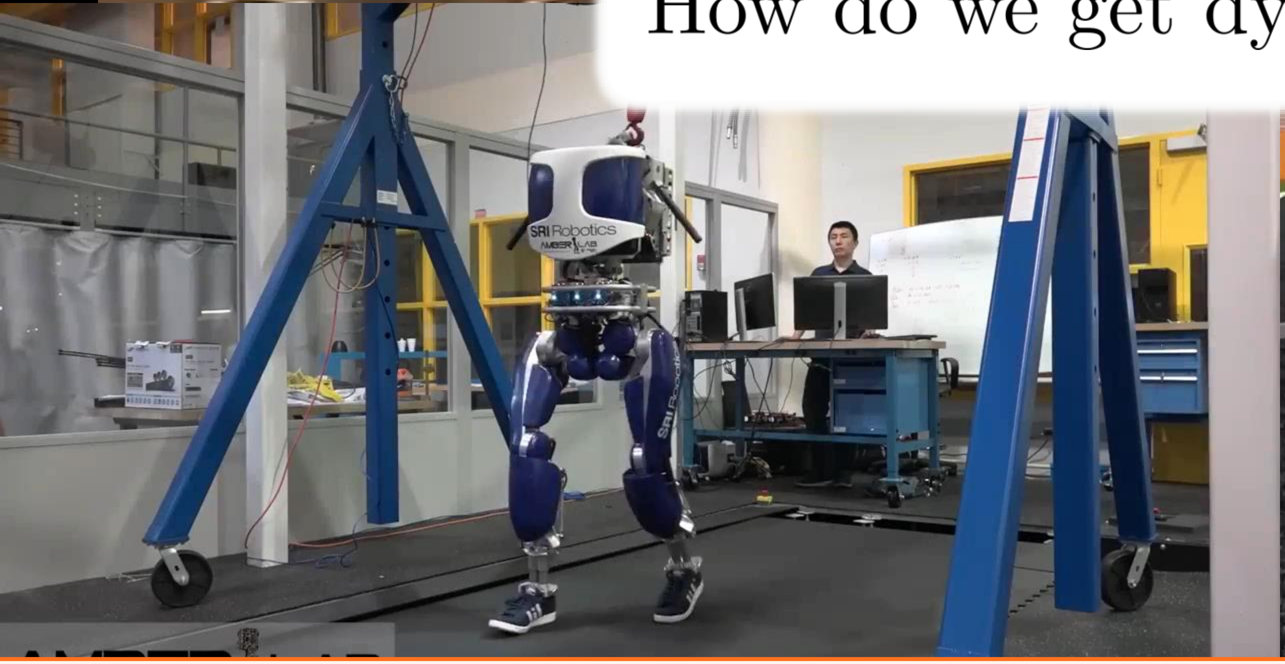
How do we get dynamic stability?



How do we achieve some task?

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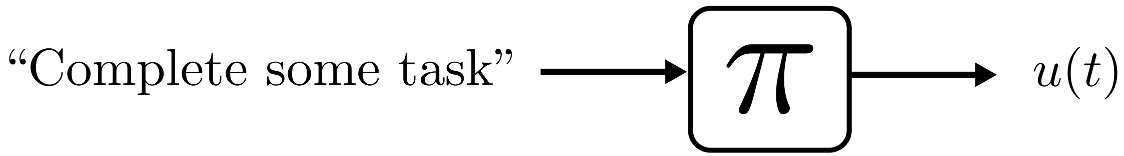
Problem Setting



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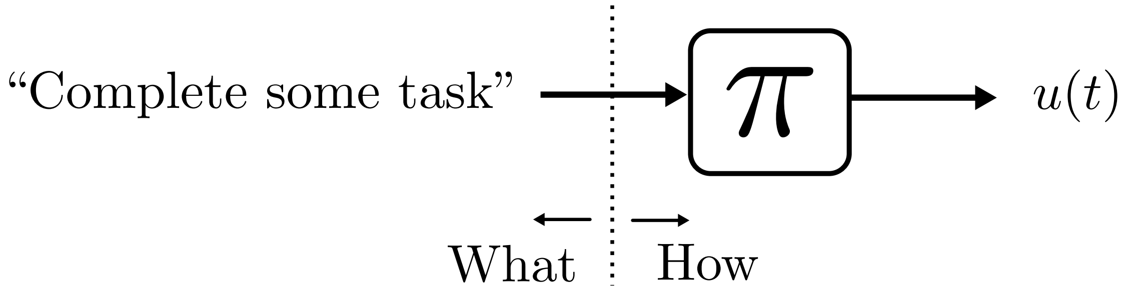
Problem Setting



A. Garg, "Building Blocks of Generalizable Autonomy: Duality of Discovery & Bias," 2022.

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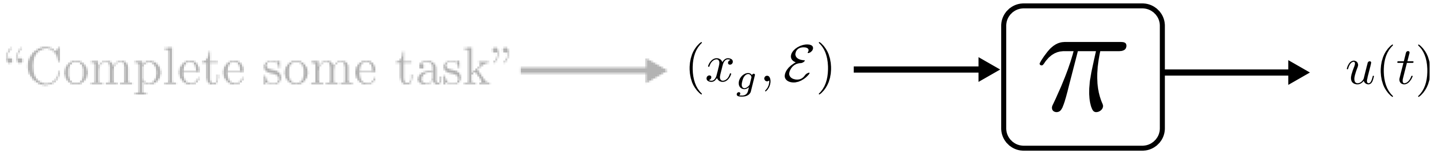
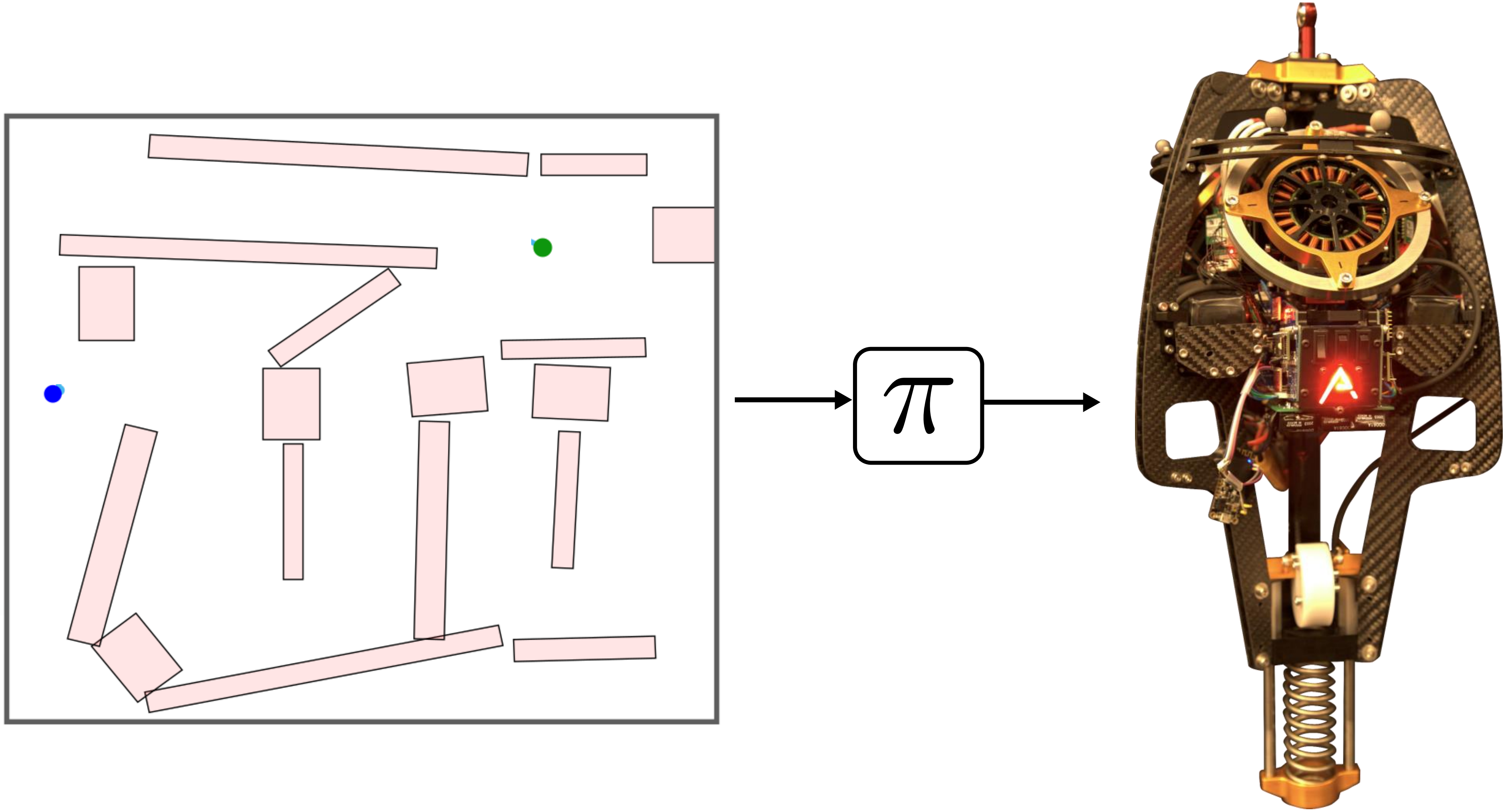
Problem Setting

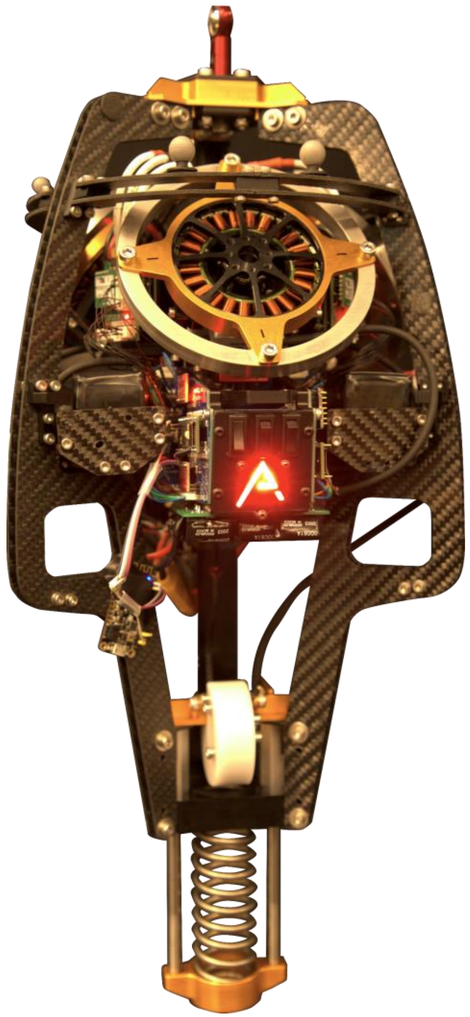


Example: 3D Hopping Robot



Example: 3D Hopping Robot

"Complete some task" → $(x_g, \mathcal{E})$ → π →



Configuration Space:

- $\mathbf{q} \in SE(3) \times \mathbb{R}^4$

Input:

- $\mathbf{u} \in \mathbb{R}^4$
- 3 flywheels for orientation control
- Pulley for foot spring compression

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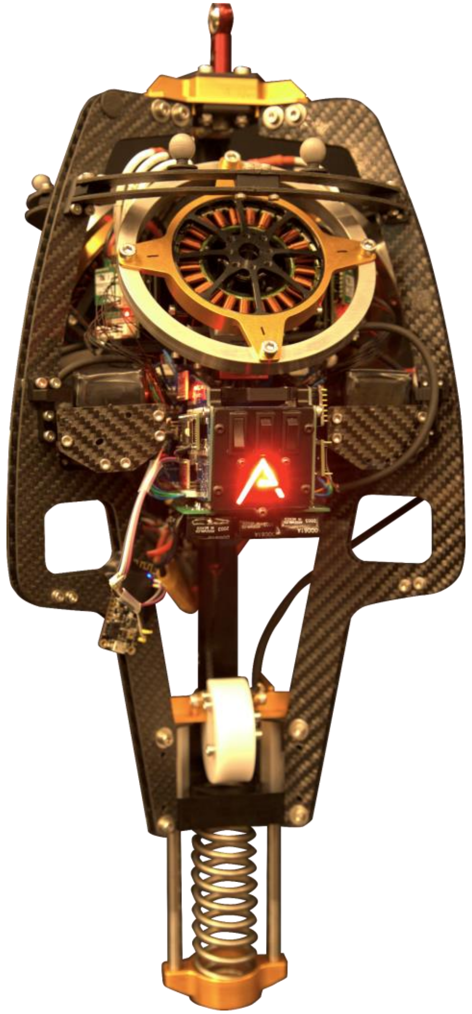
Example: 3D Hopping Robot

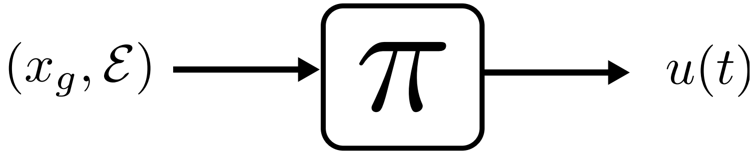
“Complete some task” → $(x_g, \mathcal{E})$ → π →



Challenges:

- Nonconvex state constraints
- Long, highly underactuated flight phases
- Relatively large dimensionality (20)
- Hybrid, nonlinear dynamics
- Manifold-valued states

A Hierarchical Approach



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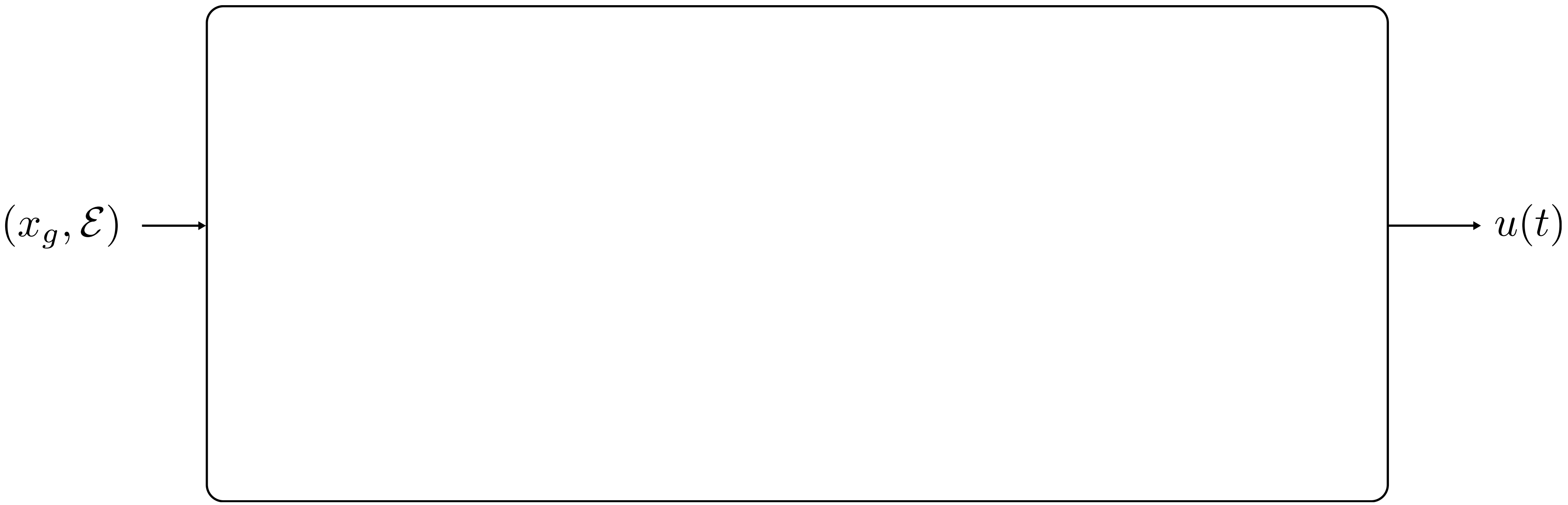

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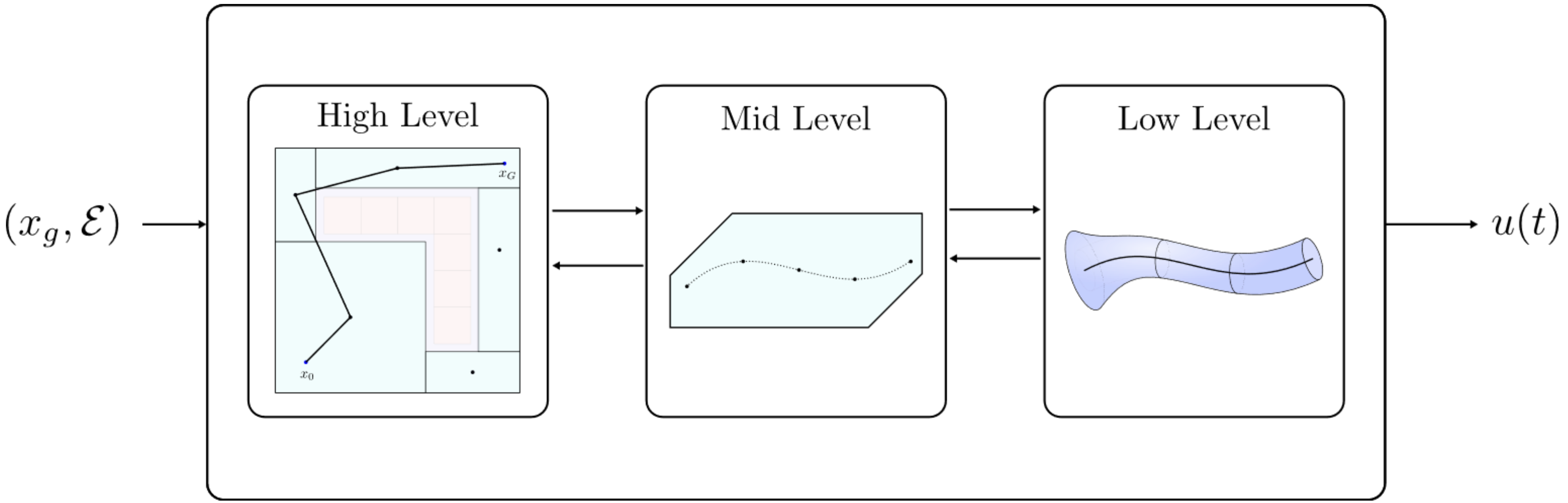

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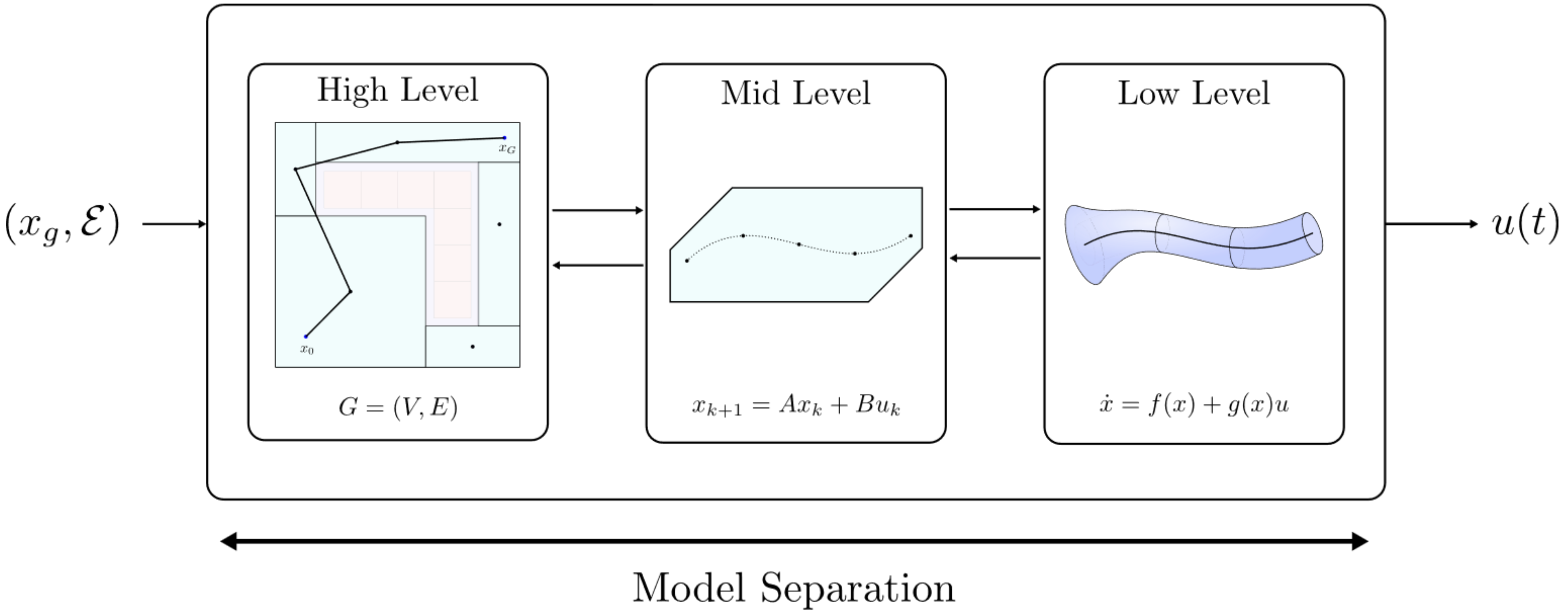
A Hierarchical Approach



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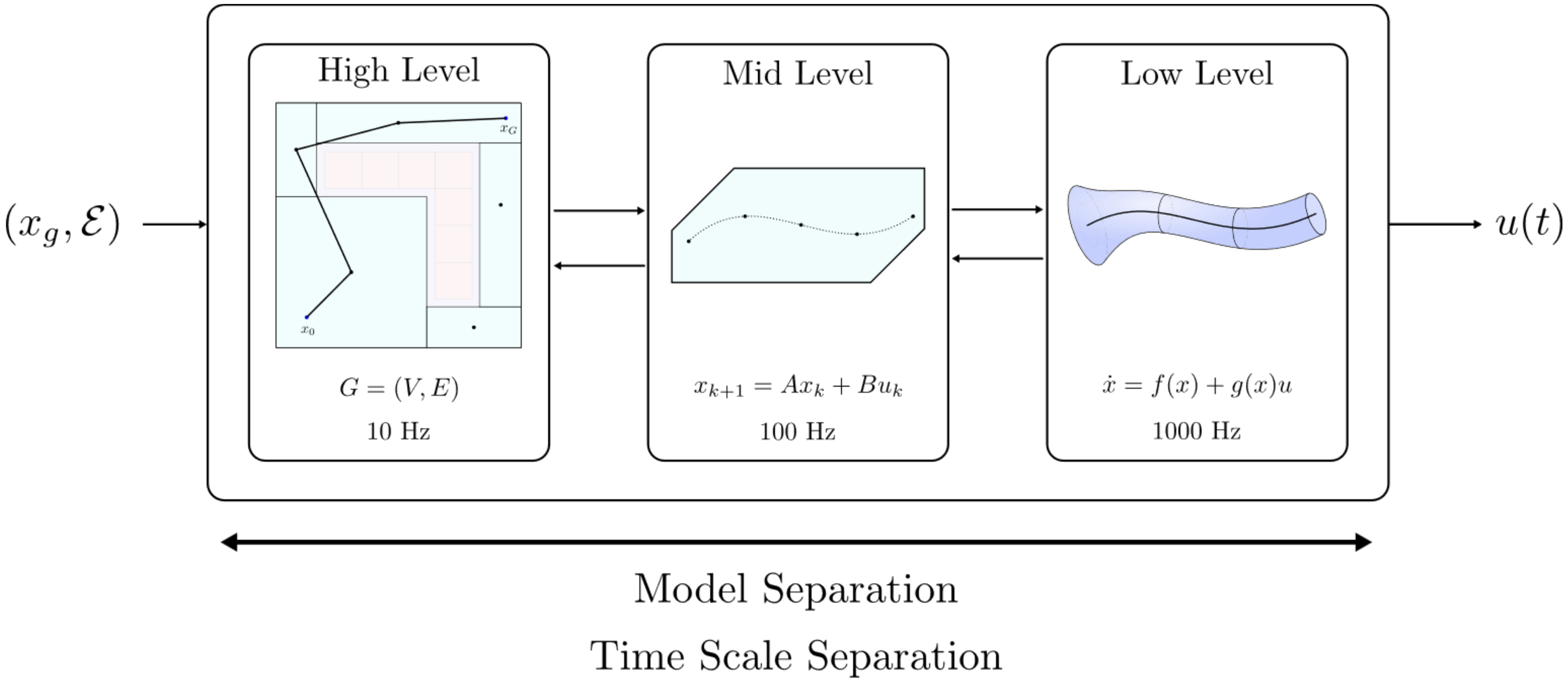

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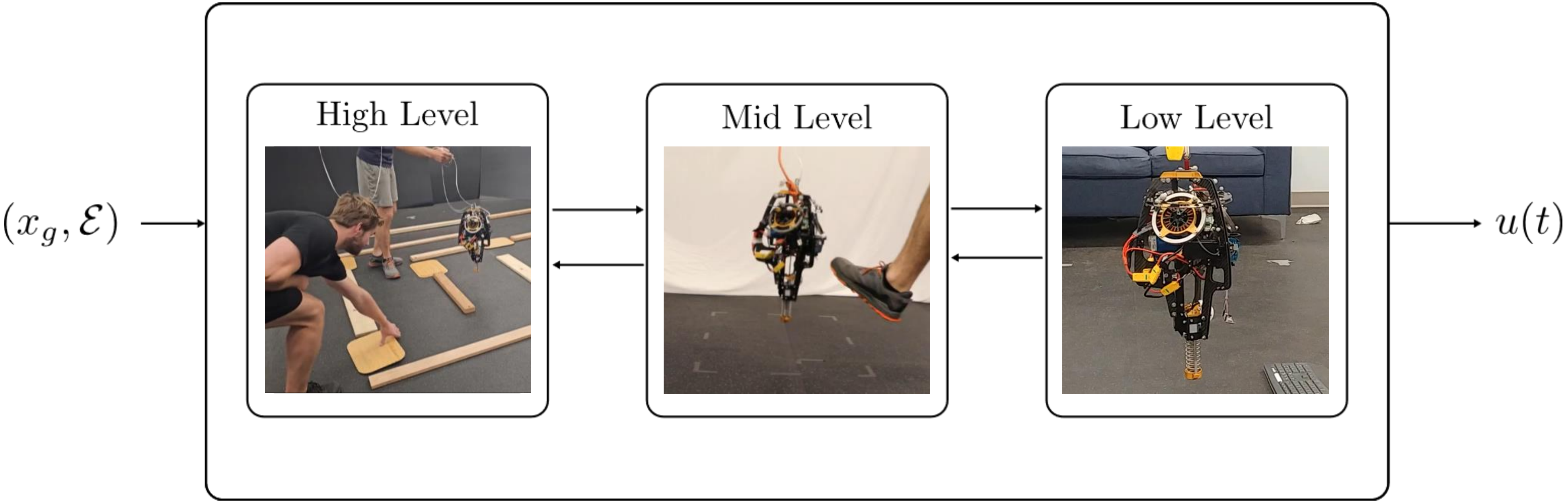
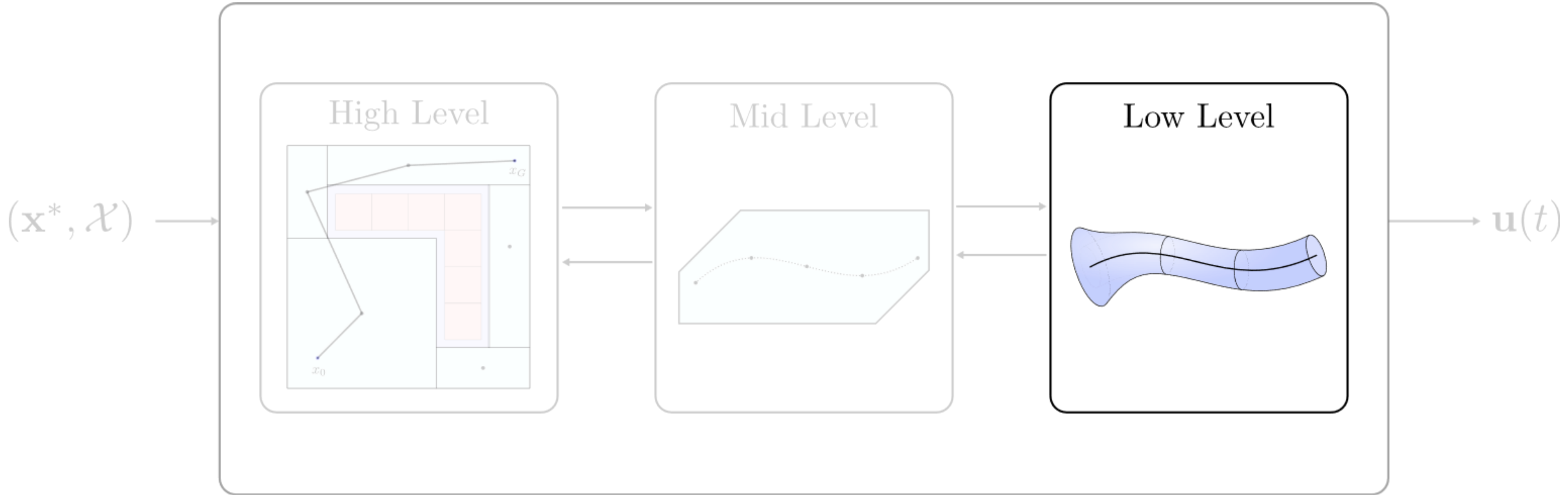


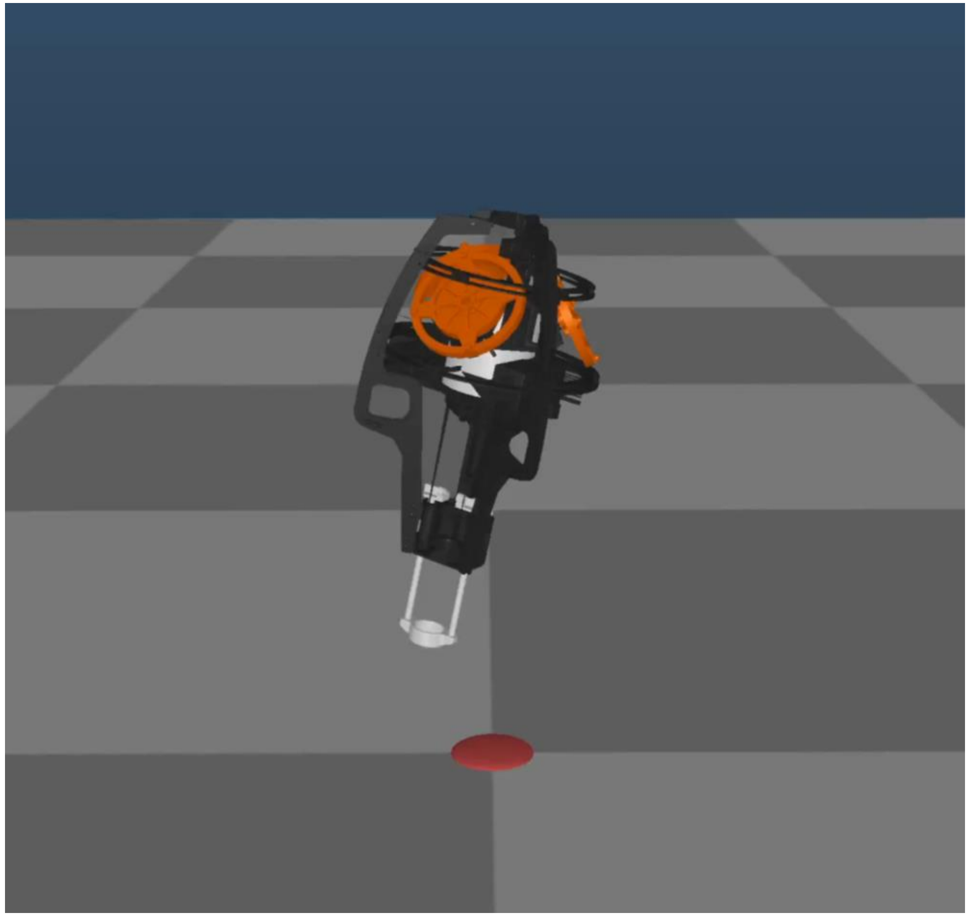
Low Level Control



ARCHER Robot

Consider the continuous-time dynamics:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{g}(\mathbf{x})\mathbf{u},$$



Controller Synthesis

Consider the continuous-time dynamics:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{g}(\mathbf{x})\mathbf{u},$$

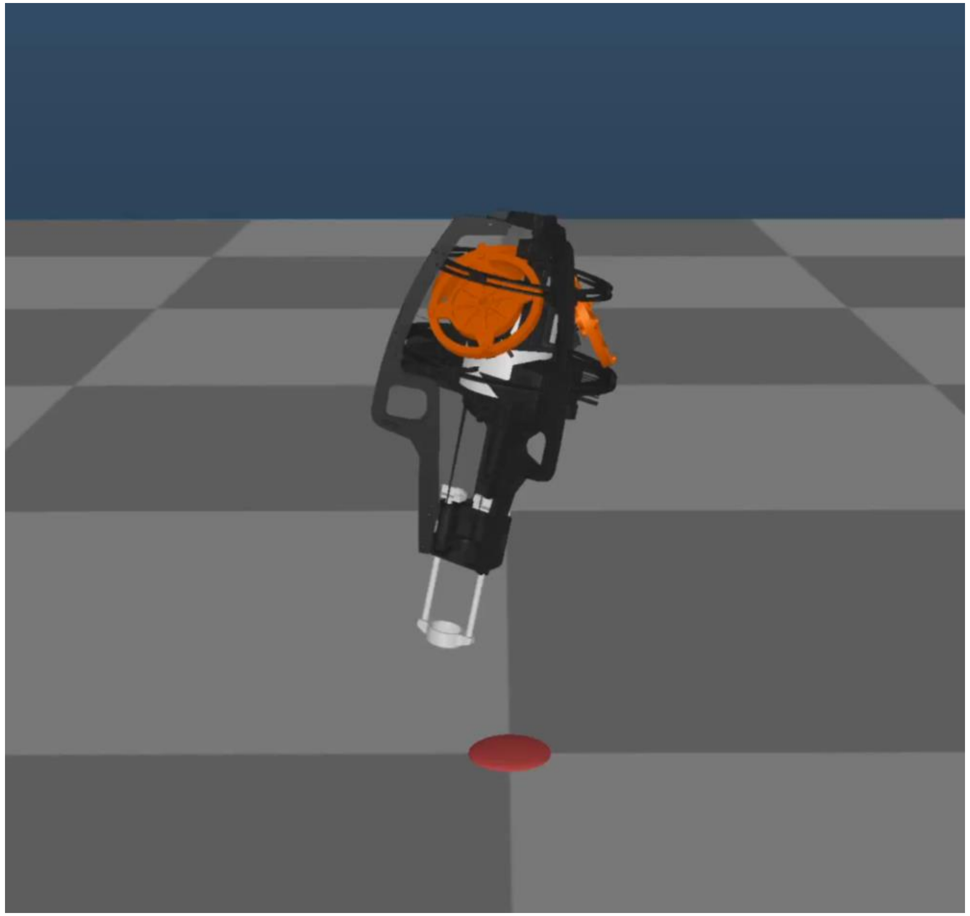
and define the outputs:

$$\mathbf{y} = \begin{bmatrix} q \\ \ell \end{bmatrix} \ominus \begin{bmatrix} q_d(t) \\ \ell_d(t) \end{bmatrix}, \qquad \boldsymbol{\eta} = \begin{bmatrix} \mathbf{y} \\ \dot{\mathbf{y}} \end{bmatrix},$$

with dynamics:

$$\dot{\boldsymbol{\eta}} = \hat{\mathbf{f}}(\boldsymbol{\eta}, \mathbf{z}) + \hat{\mathbf{g}}(\boldsymbol{\eta}, \mathbf{z})\mathbf{u}$$

Controlling the actuated states is easy.



Controller Synthesis

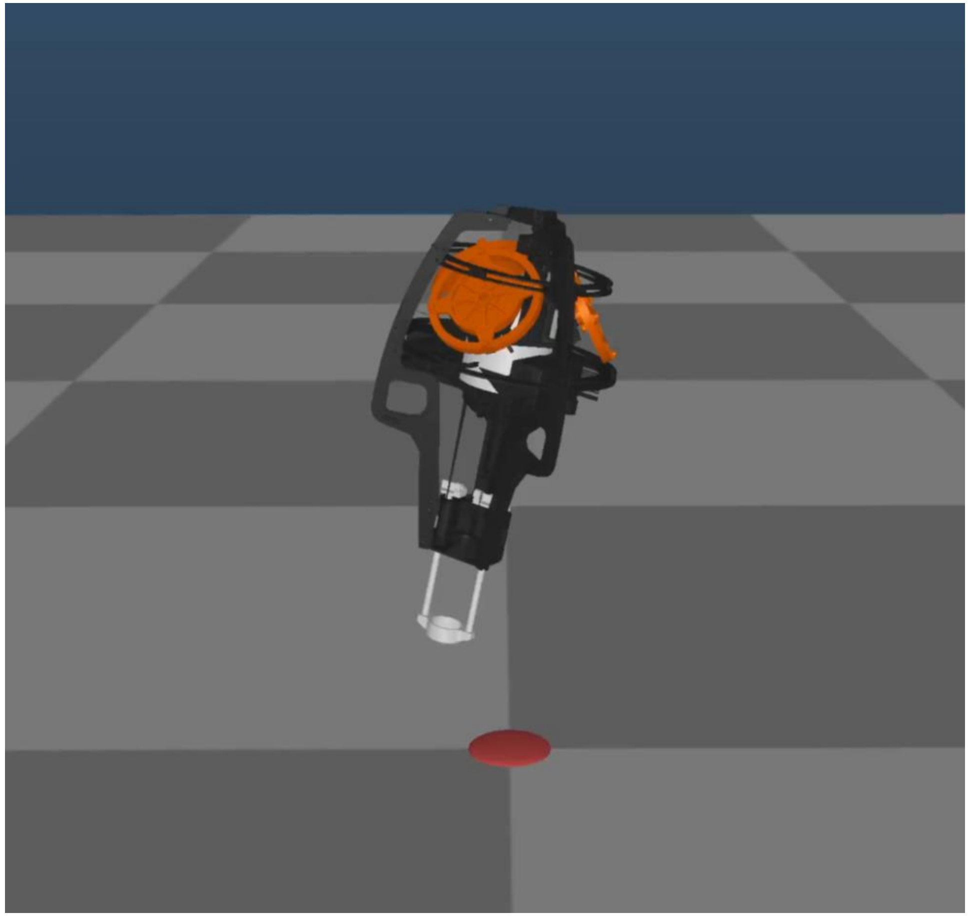
There are more than just actuated states.

We can decompose $\mathbf{x}$ into actuated and passive states:

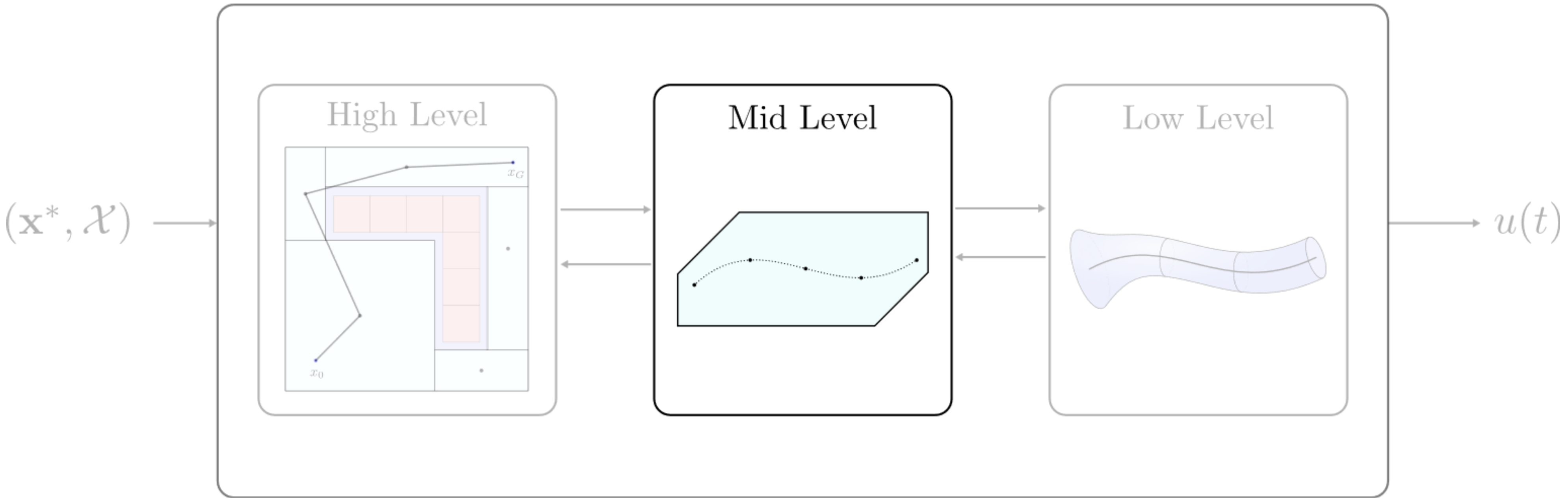
$$\dot{\boldsymbol{\eta}} = \hat{\mathbf{f}}(\boldsymbol{\eta}, \mathbf{z}) + \hat{\mathbf{g}}(\boldsymbol{\eta}, \mathbf{z})\mathbf{u}$$
$$\dot{\mathbf{z}} = \boldsymbol{\omega}(\boldsymbol{\eta}, \mathbf{z})$$

Stability in $\boldsymbol{\eta}$ and $\mathbf{z}$ $\implies$ Stability in $\mathbf{x}$.

How do we get stability in $\mathbf{z}$?

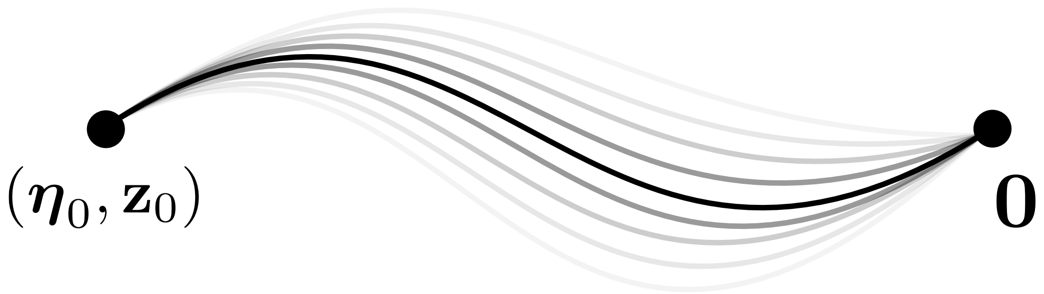


Mid Level Control



Optimal Control

$$\min_{\mathbf{u}} \int_0^\infty \hat{c}(\boldsymbol{\eta}(t), \mathbf{z}(t), \mathbf{u}(t))dt$$
$$\text{s.t.} \quad \dot{\boldsymbol{\eta}} = \hat{\mathbf{f}}(\boldsymbol{\eta}, \mathbf{z}) + \hat{\mathbf{g}}(\boldsymbol{\eta}, \mathbf{z})\mathbf{u}$$
$$\dot{\mathbf{z}} = \boldsymbol{\omega}(\boldsymbol{\eta}, \mathbf{z})$$



To get a feedback controller, there are two options:

Optimal Control

$$\min_{\mathbf{u}} \quad \int_0^\infty \hat{c}(\boldsymbol{\eta}(t), \mathbf{z}(t), \mathbf{u}(t))dt$$
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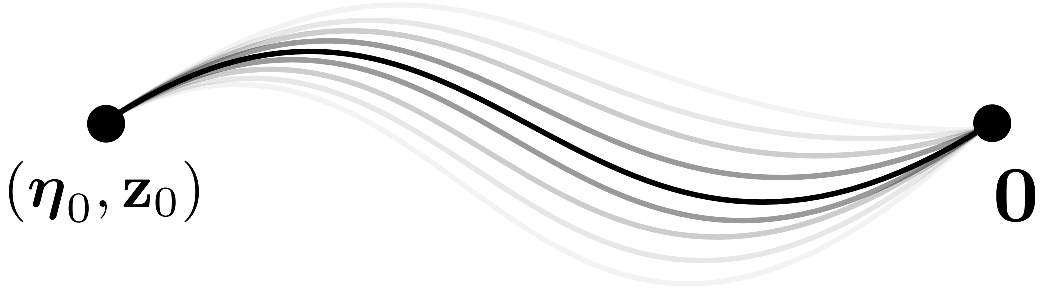
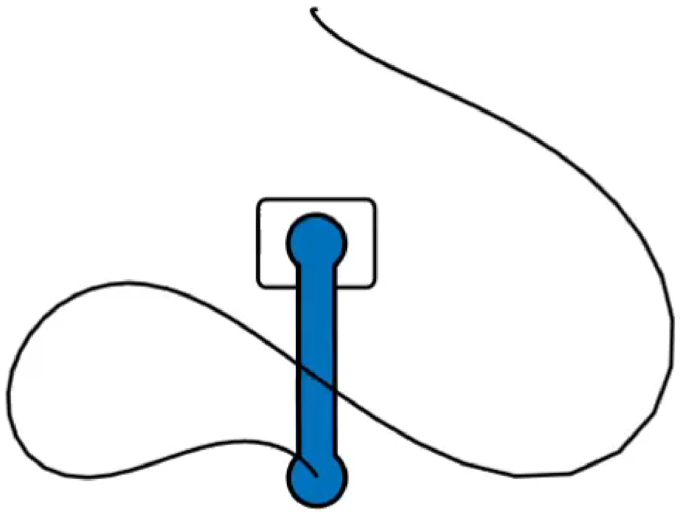
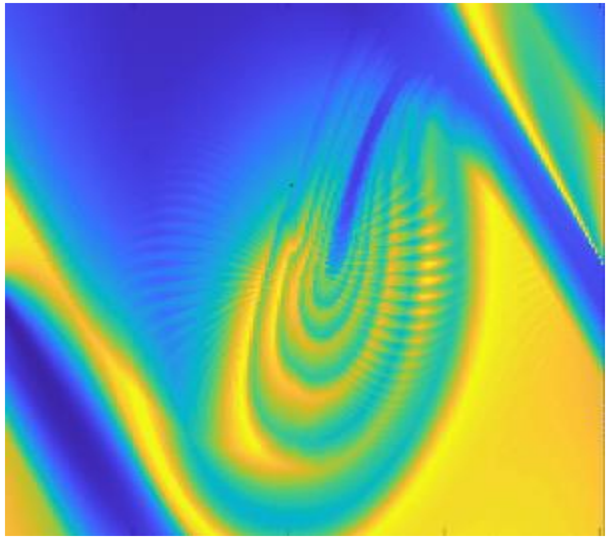


To get a feedback controller, there are two options:

Solve it Anywhere (MPC)



Solve it Everywhere (HJB)*



*This is a sampling-based approach to locally approximate the value function

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Optimal Control

$$\min_{\mathbf{u}} \quad \int_0^\infty \hat{c}(\boldsymbol{\eta}(t), \mathbf{z}(t), \mathbf{u}(t))dt$$
$$\text{s.t.} \quad \dot{\boldsymbol{\eta}} = \hat{\mathbf{f}}(\boldsymbol{\eta}, \mathbf{z}) + \hat{\mathbf{g}}(\boldsymbol{\eta}, \mathbf{z})\mathbf{u}$$
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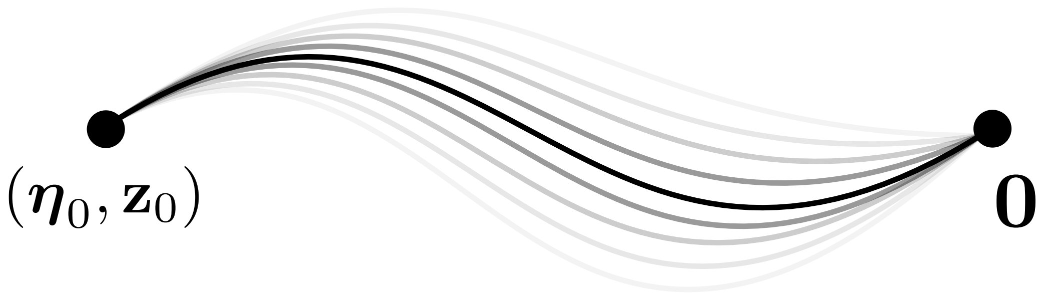

Can we leverage the $(\boldsymbol{\eta}, \mathbf{z})$ decomposition?

Optimal Control

$$\begin{aligned} \min_{\mathbf{u}} \quad & \int_0^\infty \hat{c}(\boldsymbol{\eta}(t), \mathbf{z}(t), \mathbf{u}(t))dt \\ \text{s.t.} \quad & \dot{\boldsymbol{\eta}} = \hat{\mathbf{f}}(\boldsymbol{\eta}, \mathbf{z}) + \hat{\mathbf{g}}(\boldsymbol{\eta}, \mathbf{z})\mathbf{u} \\ & \dot{\mathbf{z}} = \boldsymbol{\omega}(\boldsymbol{\eta}, \mathbf{z}) \end{aligned}$$

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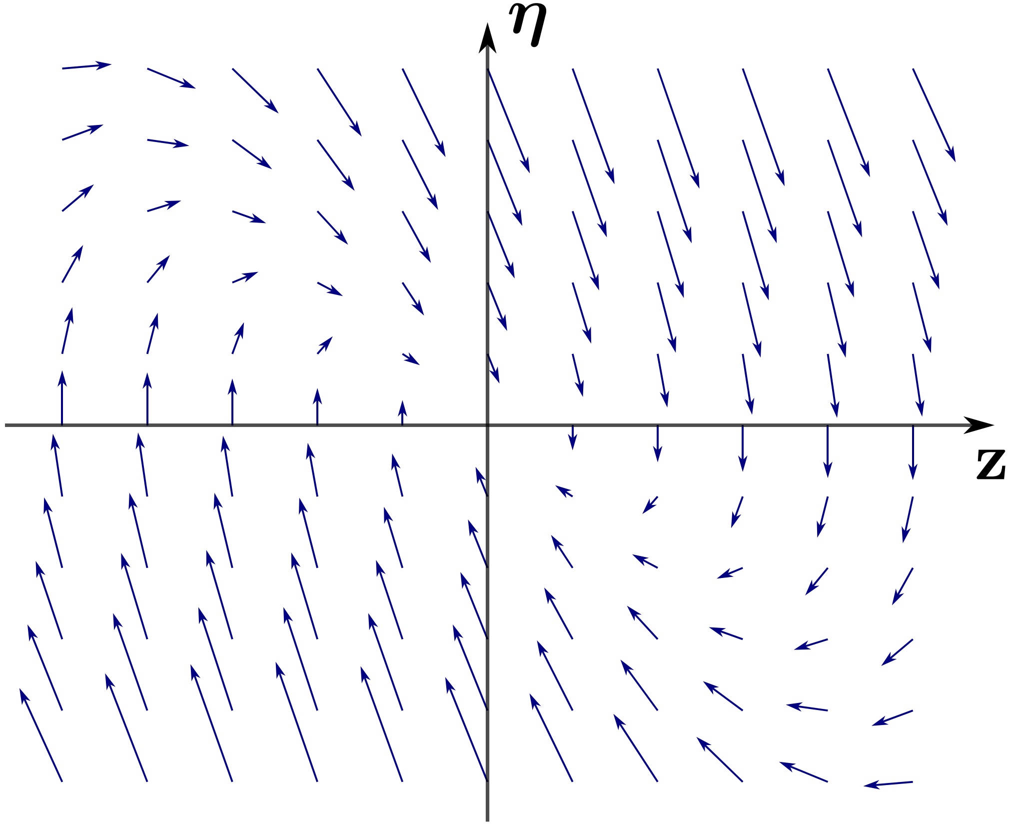

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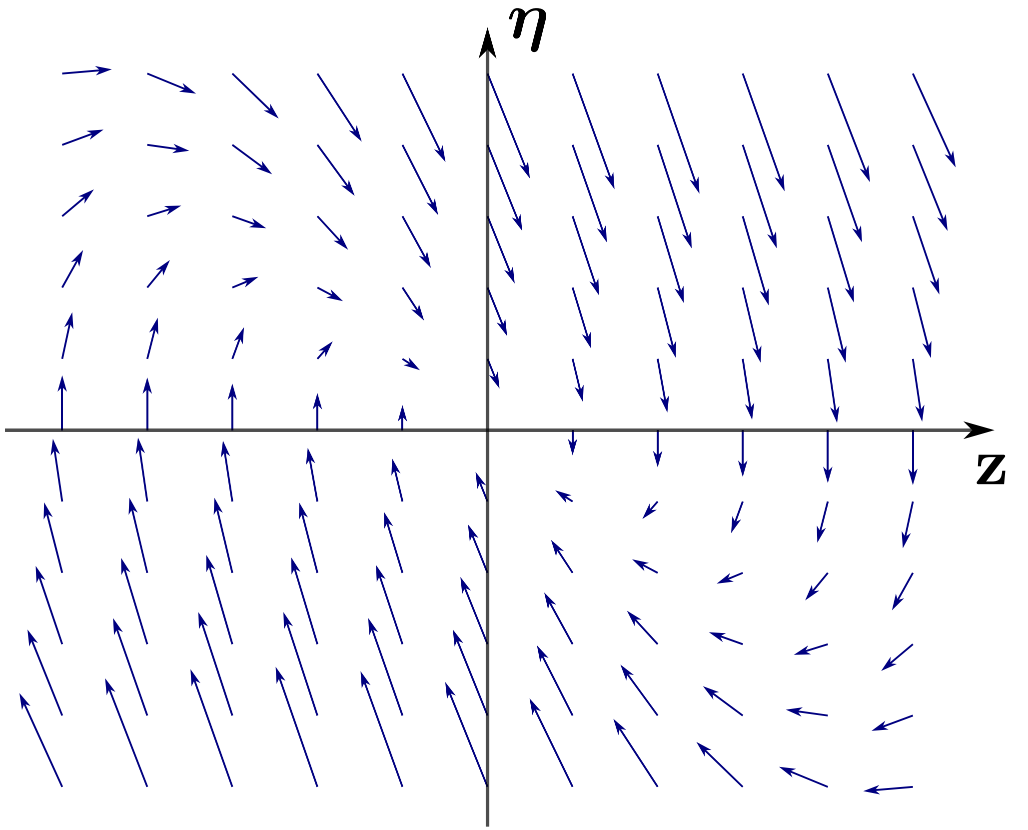
Can we leverage the $(\boldsymbol{\eta}, \mathbf{z})$ decomposition?

Find a *desired* actuated coordinate as a function of the unactuated coordinate:

$$\boldsymbol{\eta}_d = \boldsymbol{\psi}(\mathbf{z}),$$



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Find a *desired* actuated coordinate as a function of the unactuated coordinate:

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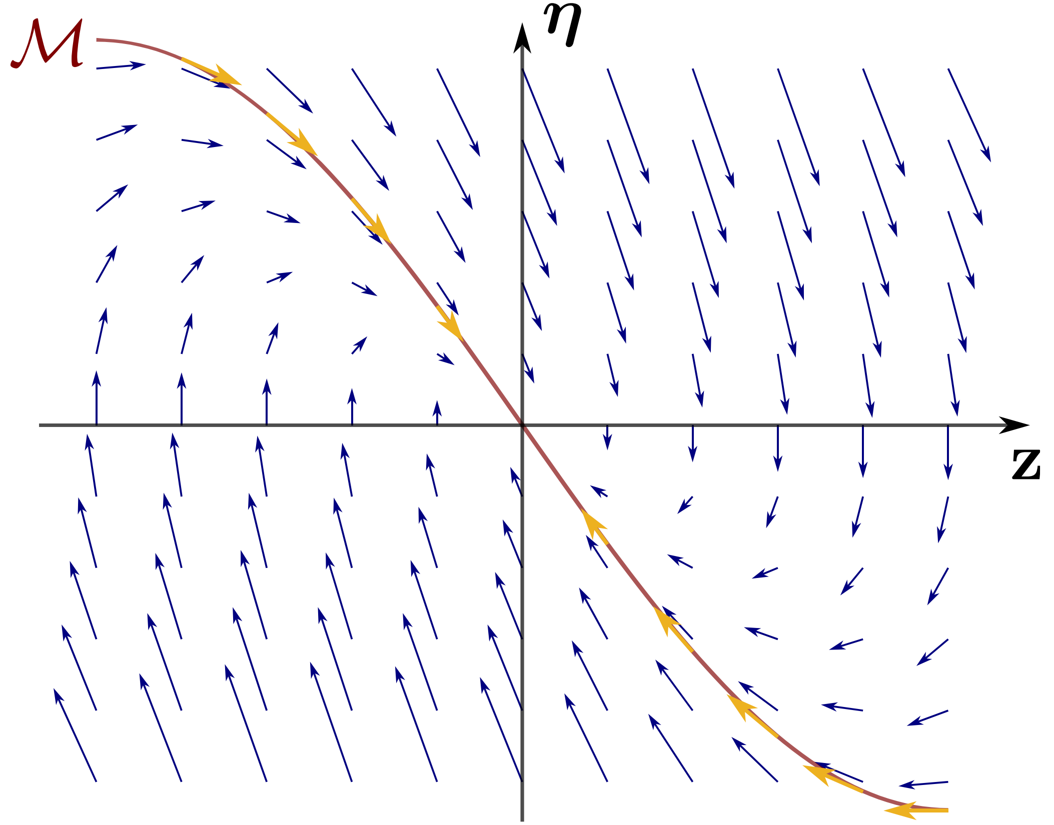
whose zeroing manifold

$$\mathcal{M} = \{(\boldsymbol{\eta}, \mathbf{z}) \mid \boldsymbol{\eta} - \boldsymbol{\psi}(\mathbf{z}) = \mathbf{0}\}$$

is invariant under optimal control.



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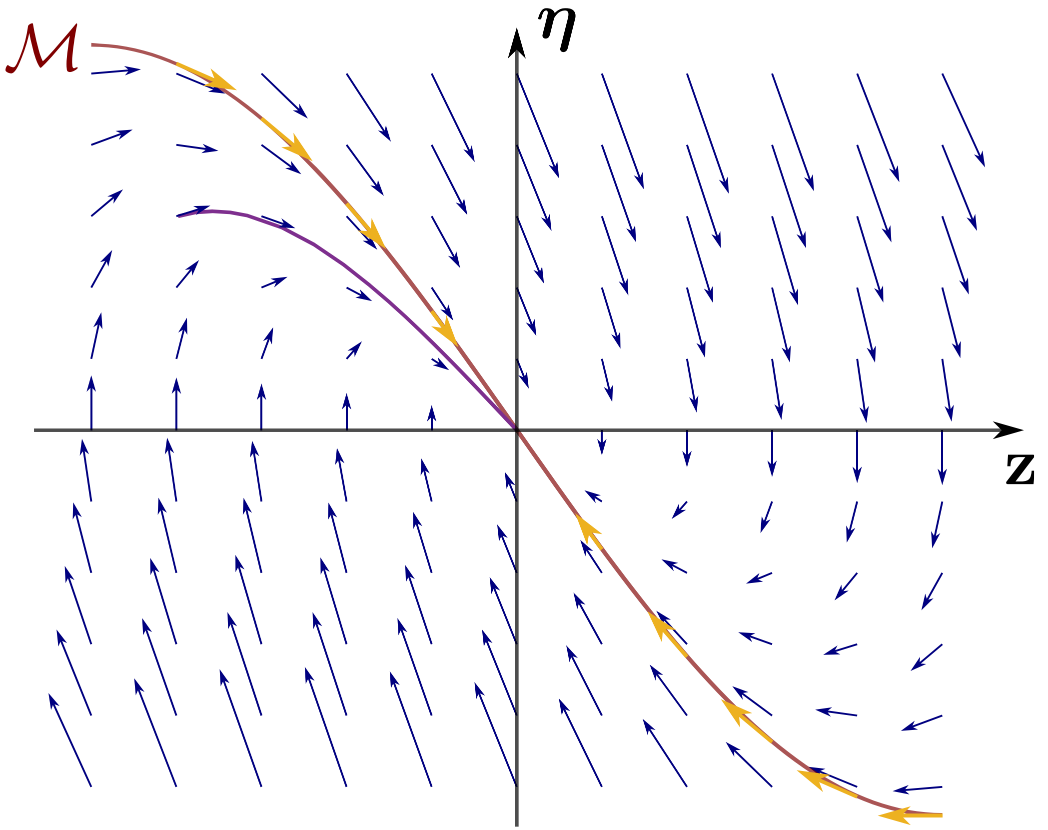
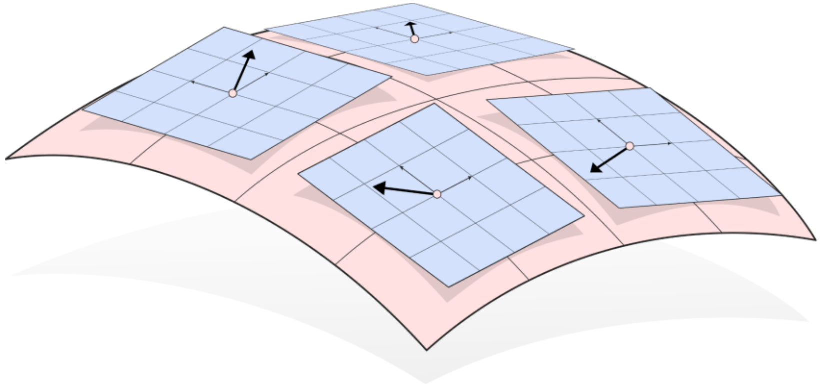
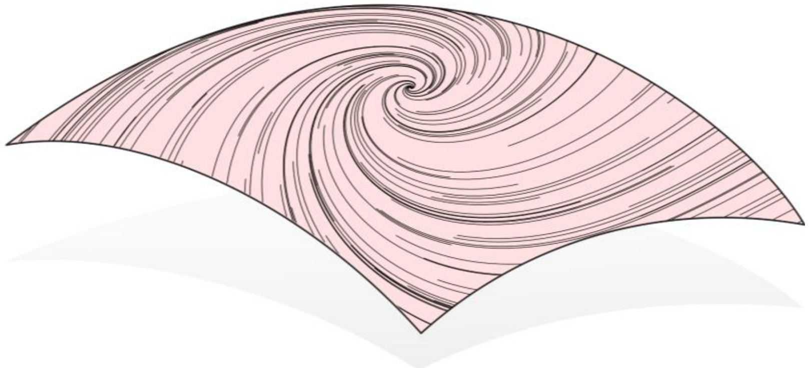


Zero Dynamics Policies

A mapping $\boldsymbol{\psi}$ with zeroing manifold $\mathcal{M} = \{(\boldsymbol{\eta}, \mathbf{z}) \mid \boldsymbol{\eta} - \boldsymbol{\psi}(\mathbf{z}) = \mathbf{0}\}$ must satisfy:



Invariance of $\mathcal{M}$



Stability of $\mathcal{M}$

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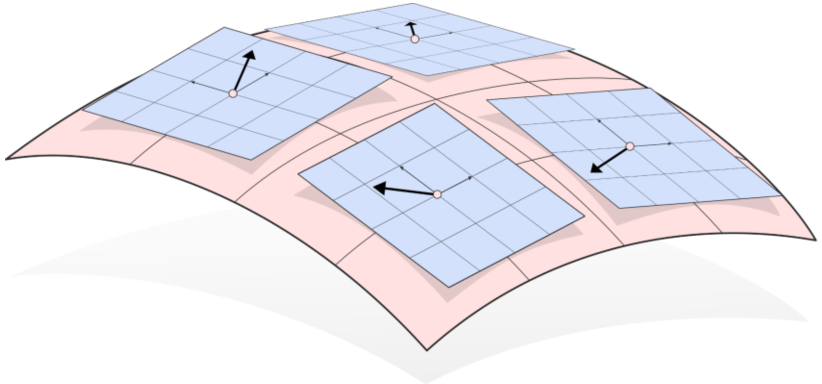

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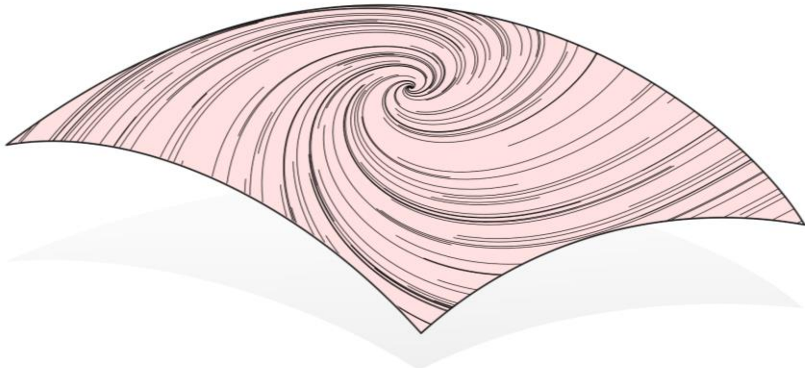

Stability of $\mathcal{M}$

$$\dot{\mathbf{x}}^* \in \mathsf{T}_{\mathbf{x}}\mathcal{M} \text{ for all } \mathbf{x} \in \mathcal{M}$$

$$\begin{aligned} \mathbf{u}^* \triangleq \arg\min_{\mathbf{u}} \quad & \int_0^\infty c(\mathbf{x}(t), \mathbf{u}(t))dt \\ \text{s.t.} \quad & \dot{\boldsymbol{\eta}} = \hat{\mathbf{f}}(\boldsymbol{\eta}, \mathbf{z}) + \hat{\mathbf{g}}(\boldsymbol{\eta}, \mathbf{z})\mathbf{u} \\ & \dot{\mathbf{z}} = \boldsymbol{\omega}(\boldsymbol{\eta}, \mathbf{z}) \end{aligned}$$

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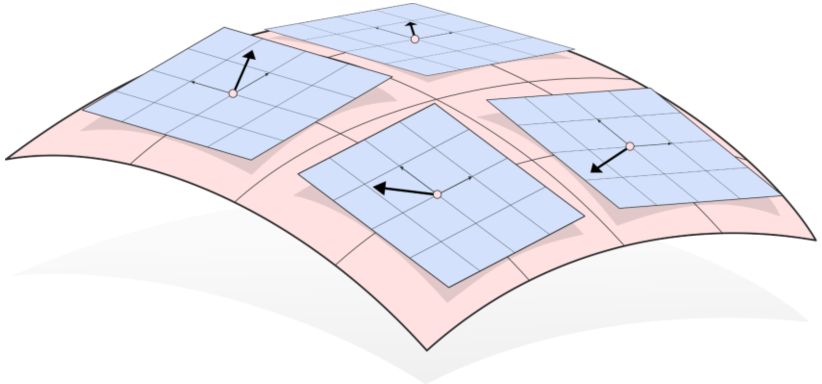

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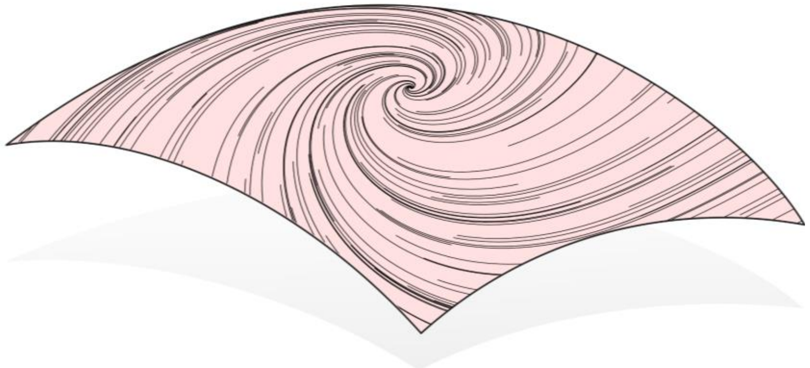

Stability of $\mathcal{M}$

This can be expressed as a loss function:

$$\mathcal{L}(\boldsymbol{\theta}) = \mathop{\mathbb{E}}_{\mathbf{z}\sim Z} \left\| \hat{\mathbf{f}}(\boldsymbol{\eta}, \mathbf{z}) + \hat{\mathbf{g}}(\boldsymbol{\eta}, \mathbf{z})\mathbf{u}^*(\boldsymbol{\eta}, \mathbf{z}) - \frac{\partial \psi_{\boldsymbol{\theta}}}{\partial \mathbf{z}} \boldsymbol{\omega}(\boldsymbol{\eta}, \mathbf{z}) \right\| \Bigg|_{\boldsymbol{\eta}=\psi_{\boldsymbol{\theta}}(\mathbf{z})}$$

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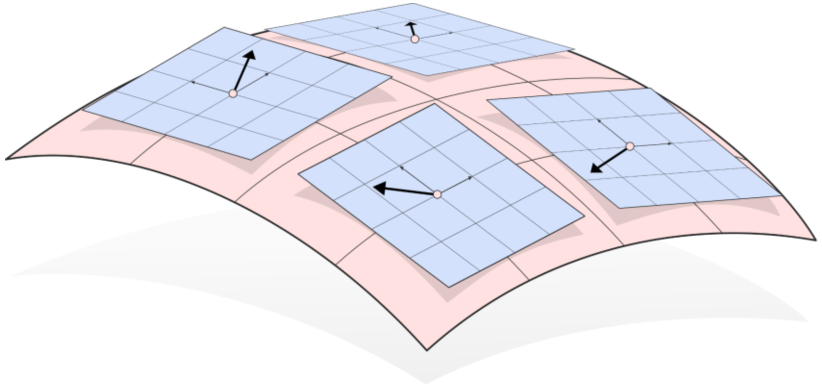

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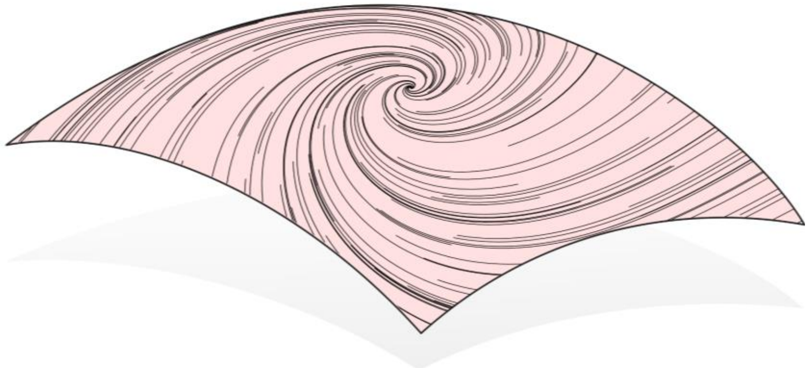

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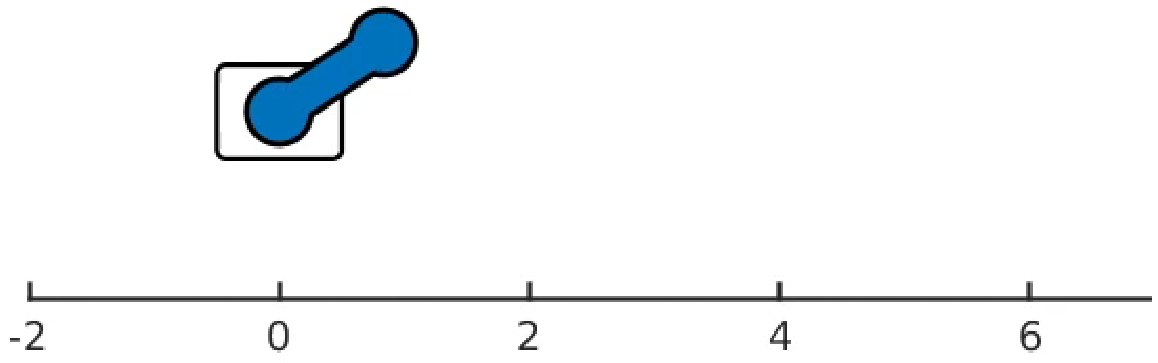
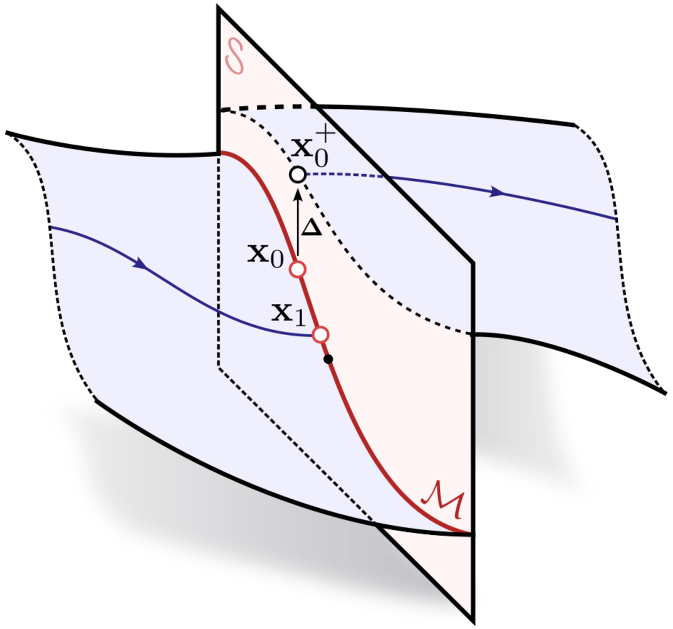
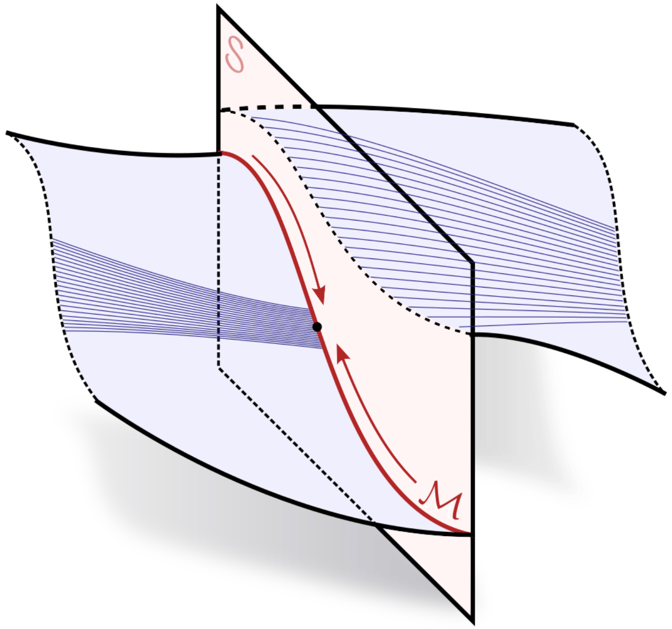


Zero Dynamics Policies for Hybrid Systems

A mapping $\boldsymbol{\psi}$ with zeroing manifold $\mathcal{M} = \{(\boldsymbol{\eta}, \mathbf{z}) \mid \boldsymbol{\eta} - \boldsymbol{\psi}(\mathbf{z}) = \mathbf{0}\}$ must satisfy:



Invariance of $\mathcal{M}$



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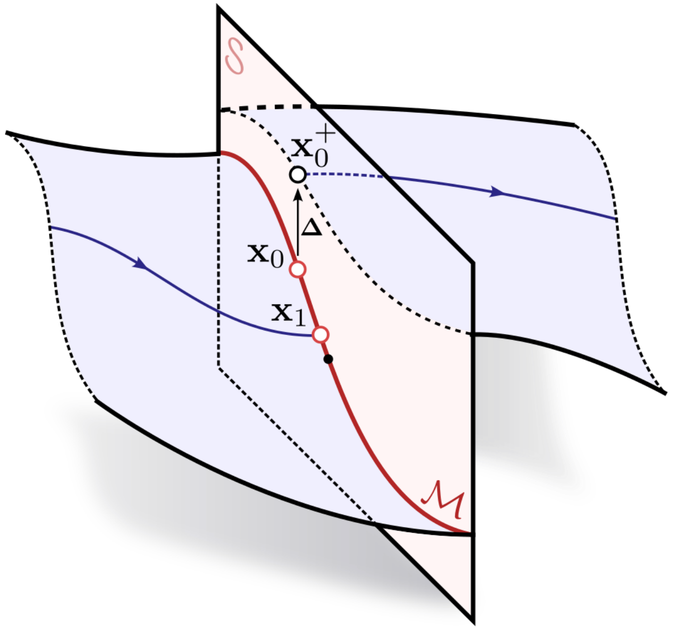
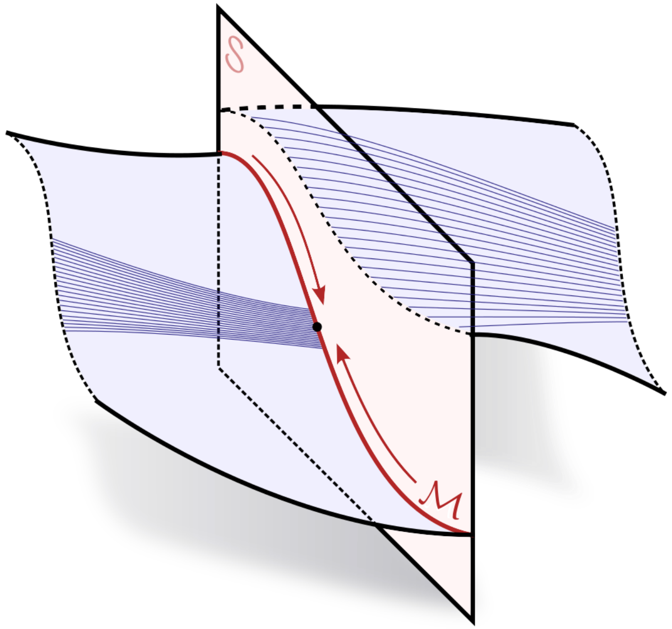


Invariance of $\mathcal{M}$

$$\mathbf{x}^*_{k+1} \in \mathcal{M} \text{ for all } \mathbf{x}_k \in \mathcal{M}$$

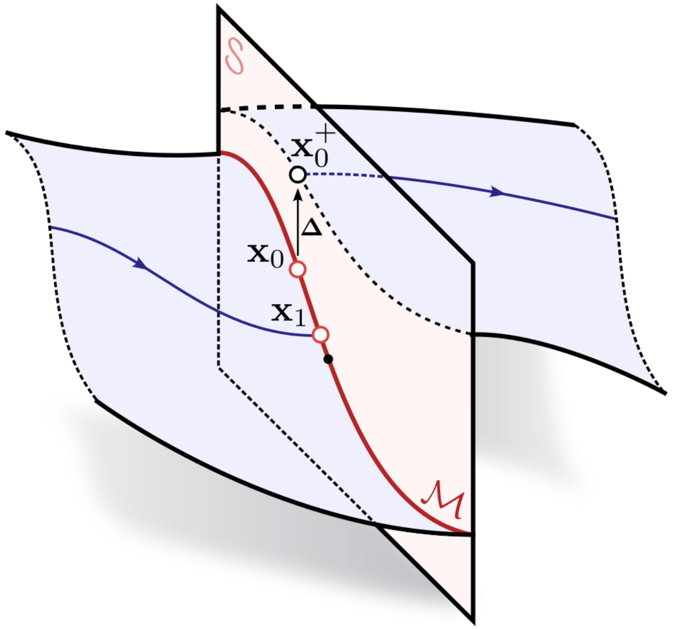


Stability of $\mathcal{M}$

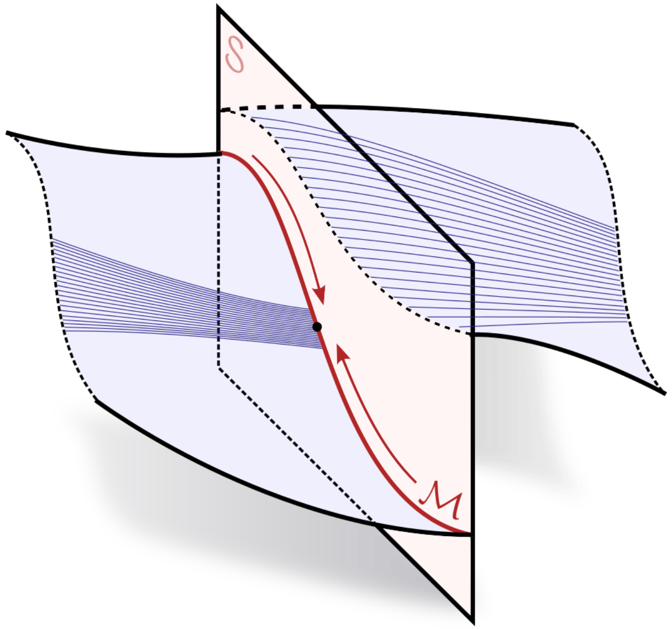
$$\mathbf{u}_0^* \triangleq \arg\min_{\mathbf{u}_k} \sum_{k=0}^{\infty} c(\mathbf{x}_k, \mathbf{u}_k)$$
$$\text{s.t.} \quad \boldsymbol{\eta}_{k+1} = \mathbf{F}(\boldsymbol{\eta}_k, \mathbf{z}_k) + \mathbf{G}(\boldsymbol{\eta}_k, \mathbf{z}_k)\mathbf{u}_k$$
$$\mathbf{z}_{k+1} = \boldsymbol{\Omega}(\boldsymbol{\eta}_k, \mathbf{z}_k)$$

Zero Dynamics Policies for Hybrid Systems

A mapping ψ with zeroing manifold $\mathcal{M} = \{(\boldsymbol{\eta}, \mathbf{z}) \mid \boldsymbol{\eta} - \boldsymbol{\psi}(\mathbf{z}) = \mathbf{0}\}$ must satisfy:



Invariance of $\mathcal{M}$



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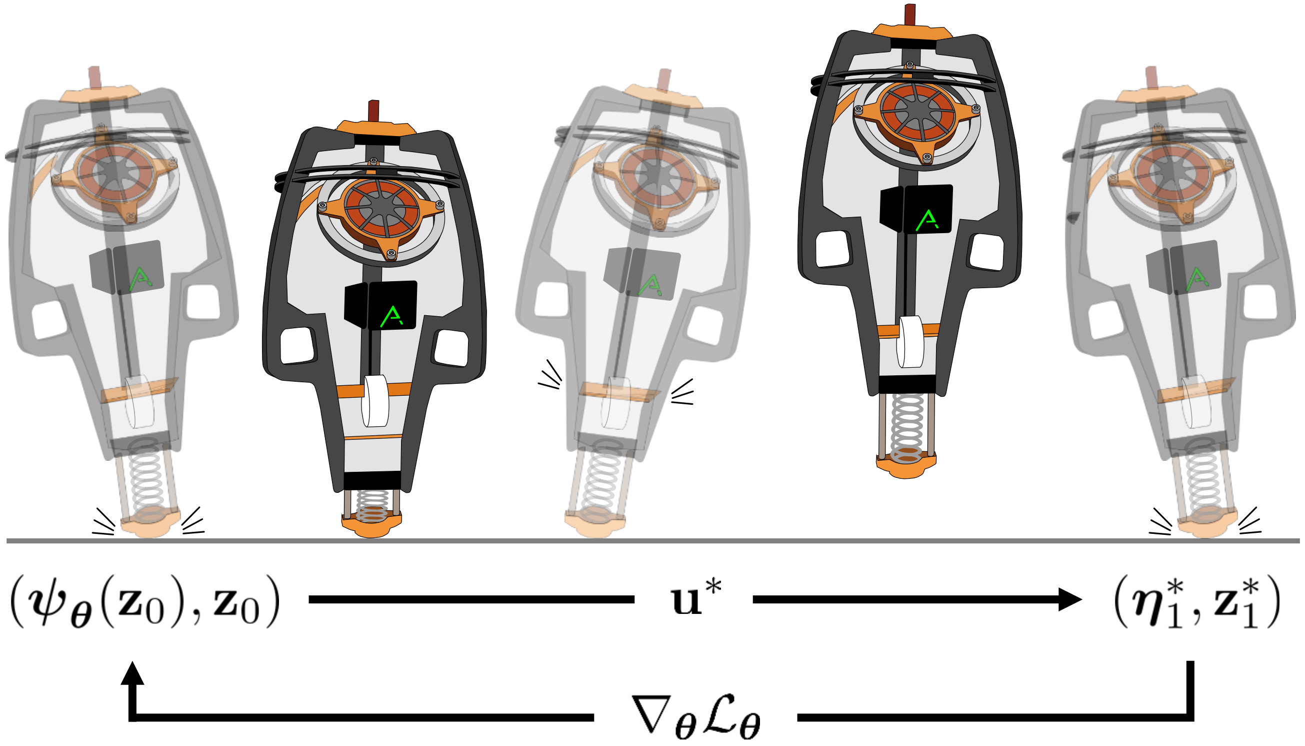
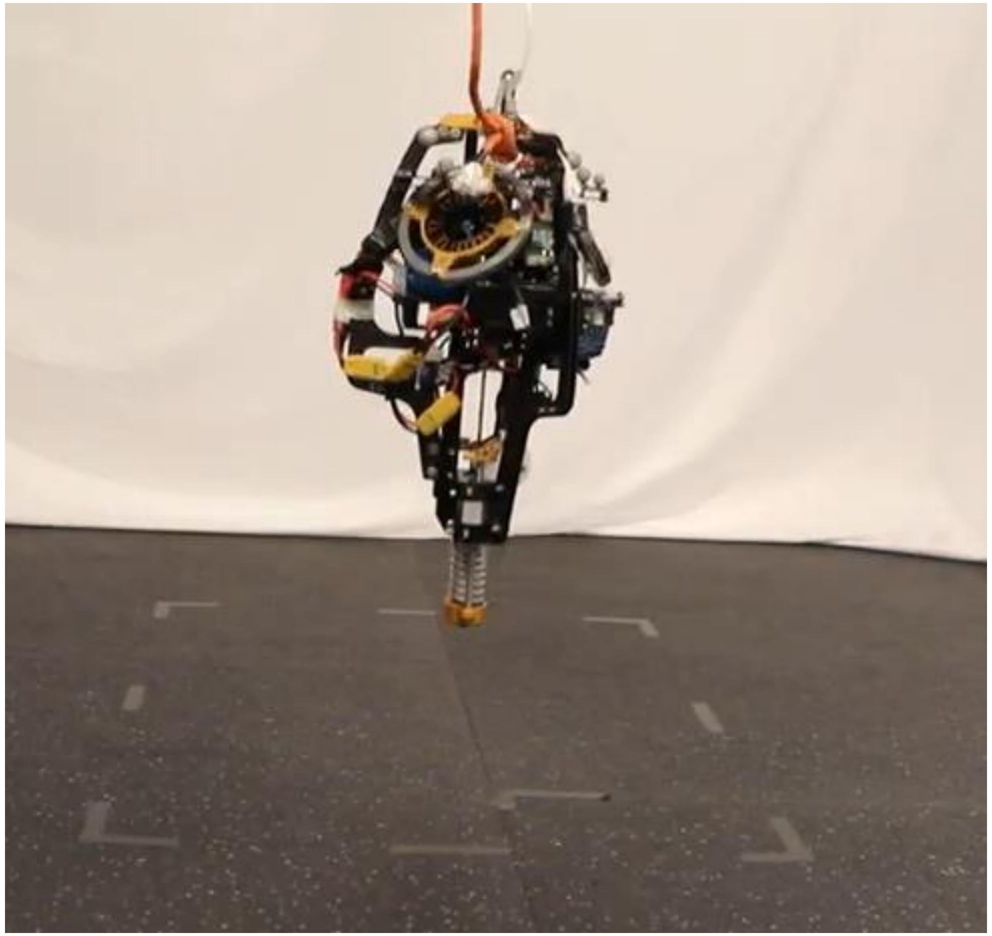
Zero Dynamics Policies for Hybrid Systems



Offline Training Procedure:

1. Sample $\mathbf{z}_0$ uniformly at impact
2. Evaluate $\psi_{\boldsymbol{\theta}}(\mathbf{z}_0)$
3. Compute $\mathbf{u}^*$ and $(\boldsymbol{\eta}_1^*, \mathbf{z}_1^*)$
4. Update $\boldsymbol{\theta}_{i+1} = \boldsymbol{\theta}_i - \rho \nabla_{\boldsymbol{\theta}} \mathcal{L}_{\boldsymbol{\theta}}$

Zero Dynamics Policies for Hybrid Systems



Online Control:

1. Evaluate $(q_d, \boldsymbol{\omega}_d) = \psi_{\boldsymbol{\theta}}(\mathbf{z})$
2. Compute output error $\mathbf{y}$:

$$\mathbf{y} = \begin{bmatrix} q \\ \ell \end{bmatrix} \ominus \begin{bmatrix} q_d \\ \ell_d \end{bmatrix}$$

3. Apply torque $\mathbf{u}$:

$$\mathbf{u} = -\begin{bmatrix} \mathbf{K}_p & \mathbf{K}_d \end{bmatrix} \begin{bmatrix} \mathbf{y} \\ \dot{\mathbf{y}} \end{bmatrix}$$

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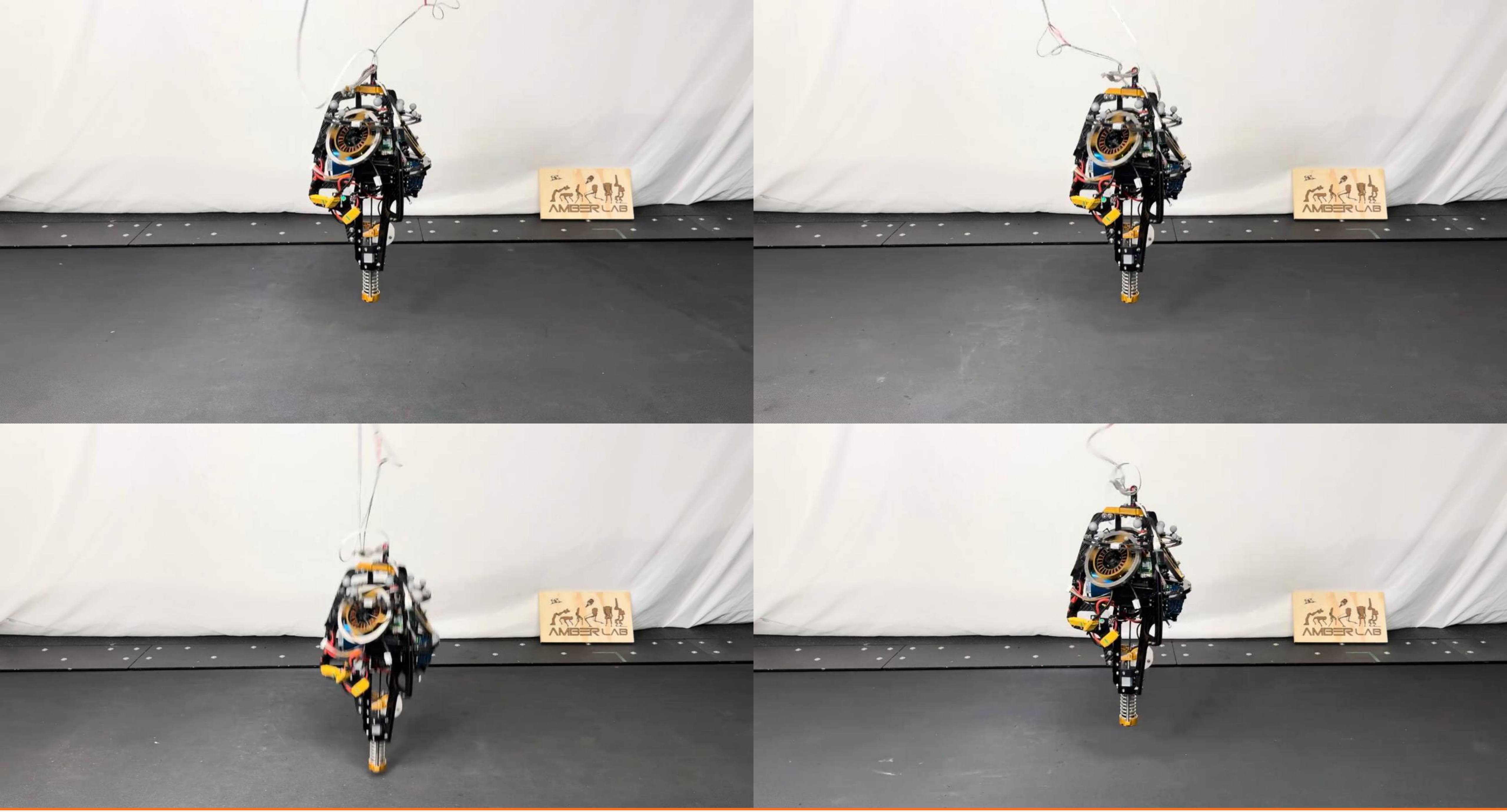
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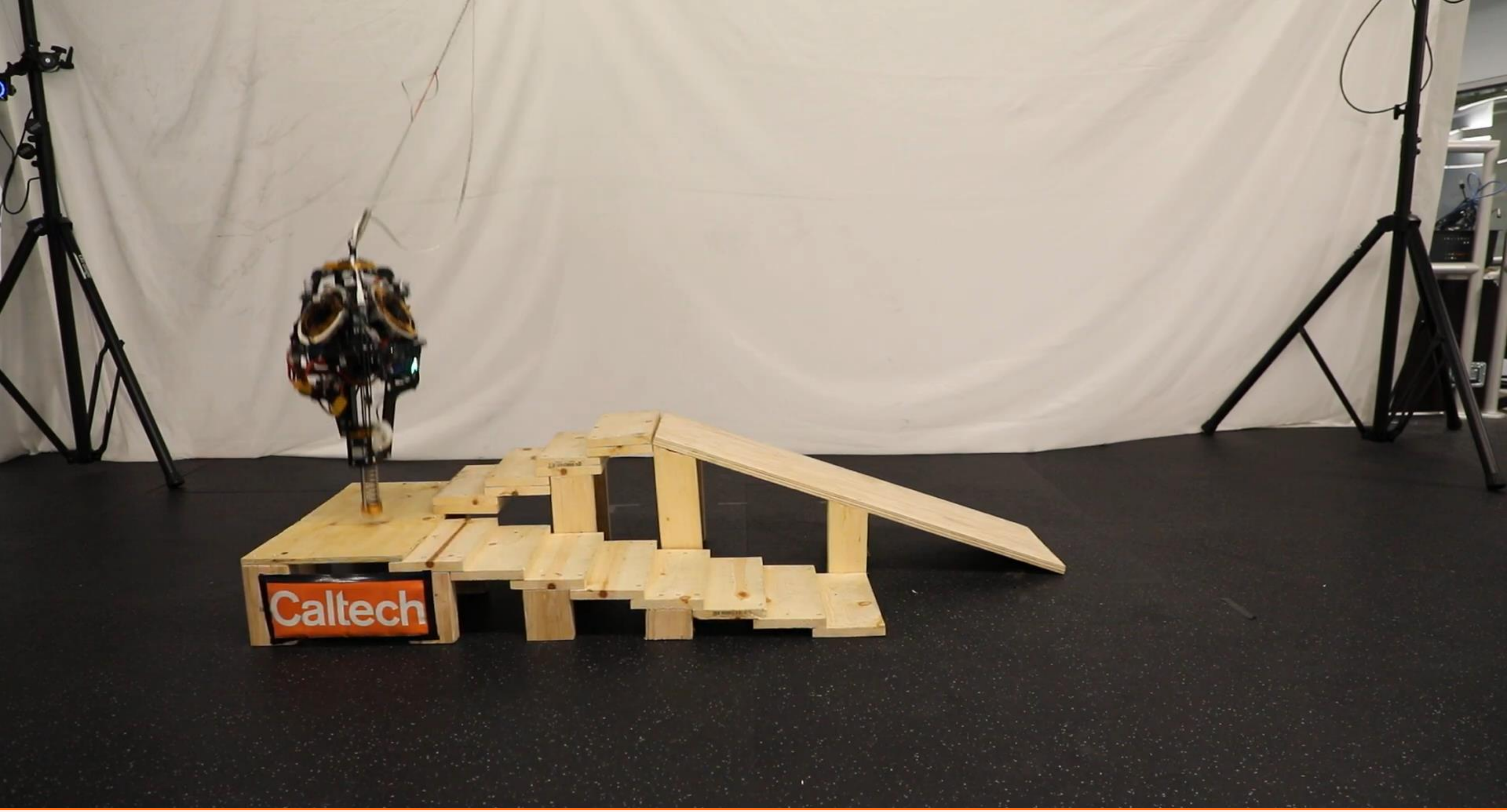
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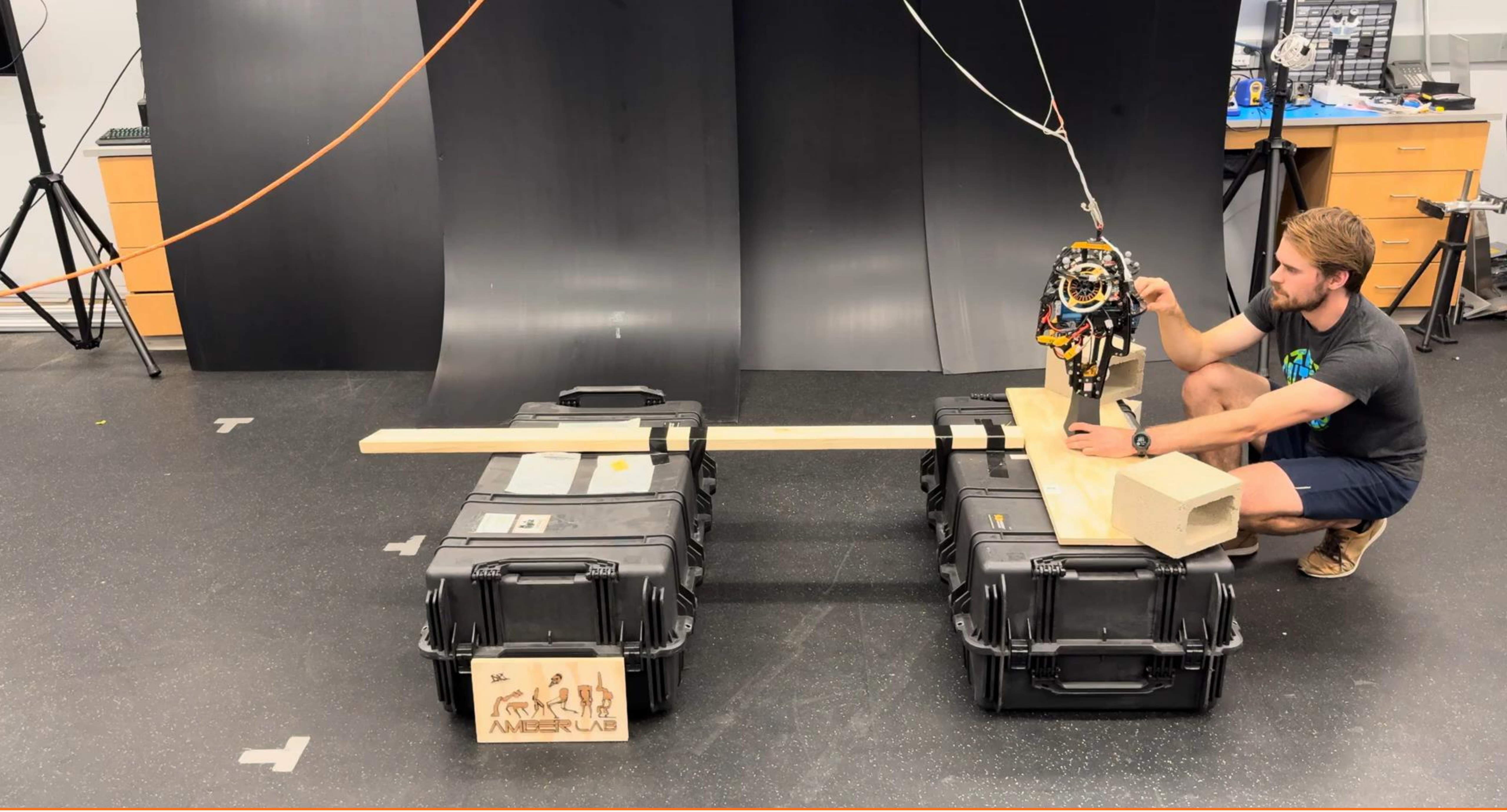
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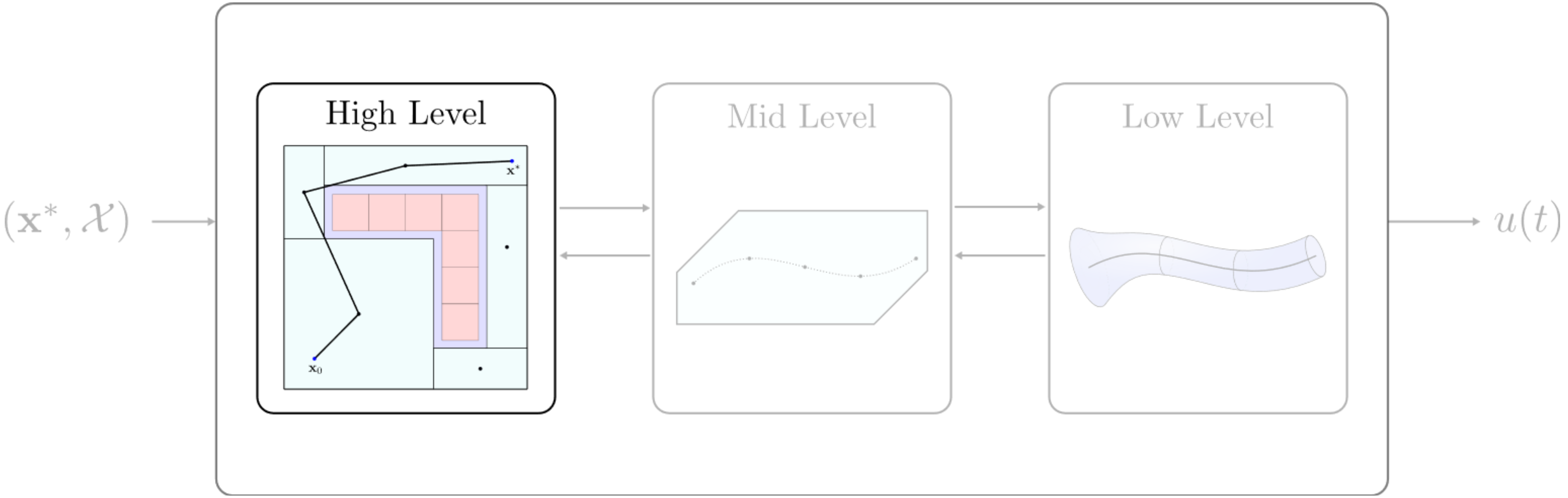
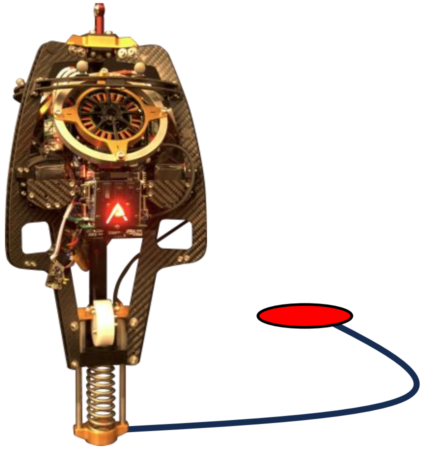
High Level Control



Given the previous constructions, the complex system:

$$\dot{\boldsymbol{\eta}} = \hat{\mathbf{f}}(\boldsymbol{\eta}, \mathbf{z}) + \hat{\mathbf{g}}(\boldsymbol{\eta}, \mathbf{z})\mathbf{u}, \quad \dot{\mathbf{z}} = \boldsymbol{\omega}(\boldsymbol{\eta}, \mathbf{z}), \quad \boldsymbol{\Phi}^{-1}(\boldsymbol{\eta}, \mathbf{z}) \notin \mathcal{S}$$

$$\boldsymbol{\eta}^+ = \boldsymbol{\Delta}_{\boldsymbol{\eta}}(\boldsymbol{\eta}^-, \mathbf{z}^-), \quad \mathbf{z}^+ = \boldsymbol{\Delta}_{\mathbf{z}}(\boldsymbol{\eta}^-, \mathbf{z}^-), \quad \boldsymbol{\Phi}^{-1}(\boldsymbol{\eta}, \mathbf{z}) \in \mathcal{S}$$



$\mathbf{x} \in \mathcal{X} \subset \mathbb{R}^{20}$

$\mathbf{u} \in \mathcal{U} \subset \mathbb{R}^{4}$

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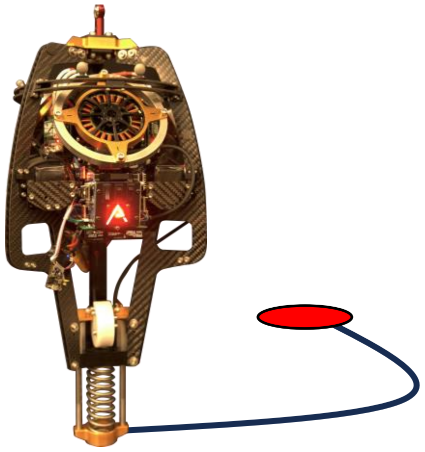
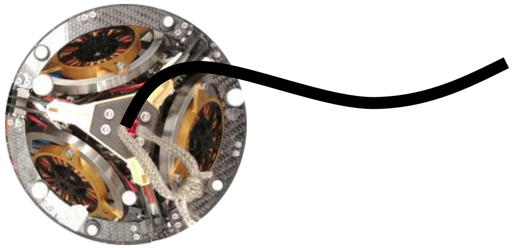


$\mathbf{x} \in \mathcal{X} \subset \mathbb{R}^{20}$

$\mathbf{u} \in \mathcal{U} \subset \mathbb{R}^{3}$

Can be abstracted as a simple system:

$$\dot{\mathbf{x}}_d = \begin{bmatrix} \mathbf{0} & \mathbf{I}_{n-m} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \mathbf{x}_d + \begin{bmatrix} \mathbf{0} \\ \mathbf{I} \end{bmatrix} \mathbf{u}_d$$



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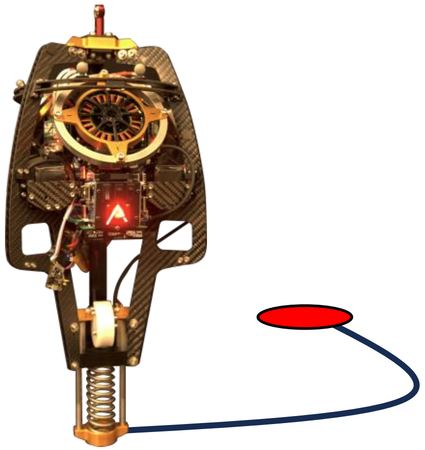

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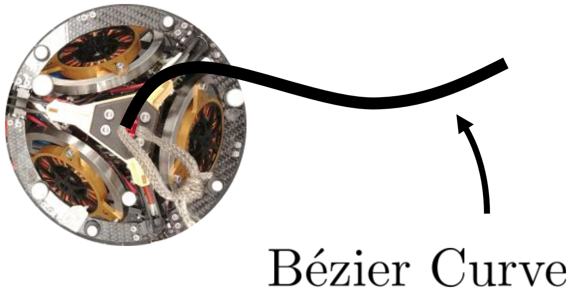

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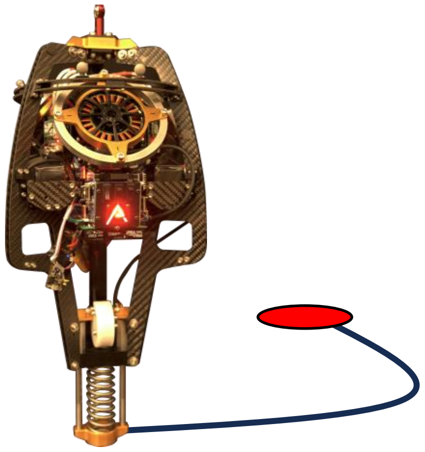
$$\boldsymbol{\eta}^{+} = \boldsymbol{\Delta}_{\boldsymbol{\eta}}(\boldsymbol{\eta}^{-},\mathbf{z}^{-}), \quad \mathbf{z}^{+} = \boldsymbol{\Delta}_{\mathbf{z}}(\boldsymbol{\eta}^{-},\mathbf{z}^{-}), \quad \boldsymbol{\Phi}^{-1}(\boldsymbol{\eta},\mathbf{z}) \in \mathcal{S}$$



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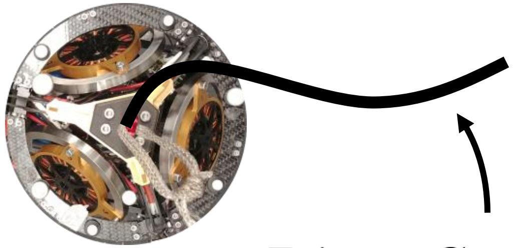


Bézier Curve

$\mathbf{x}_d \in \mathcal{X}_d \subset \mathbb{R}^{4}$

$\mathbf{u}_d \in \mathcal{U}_d \subset \mathbb{R}^{2}$

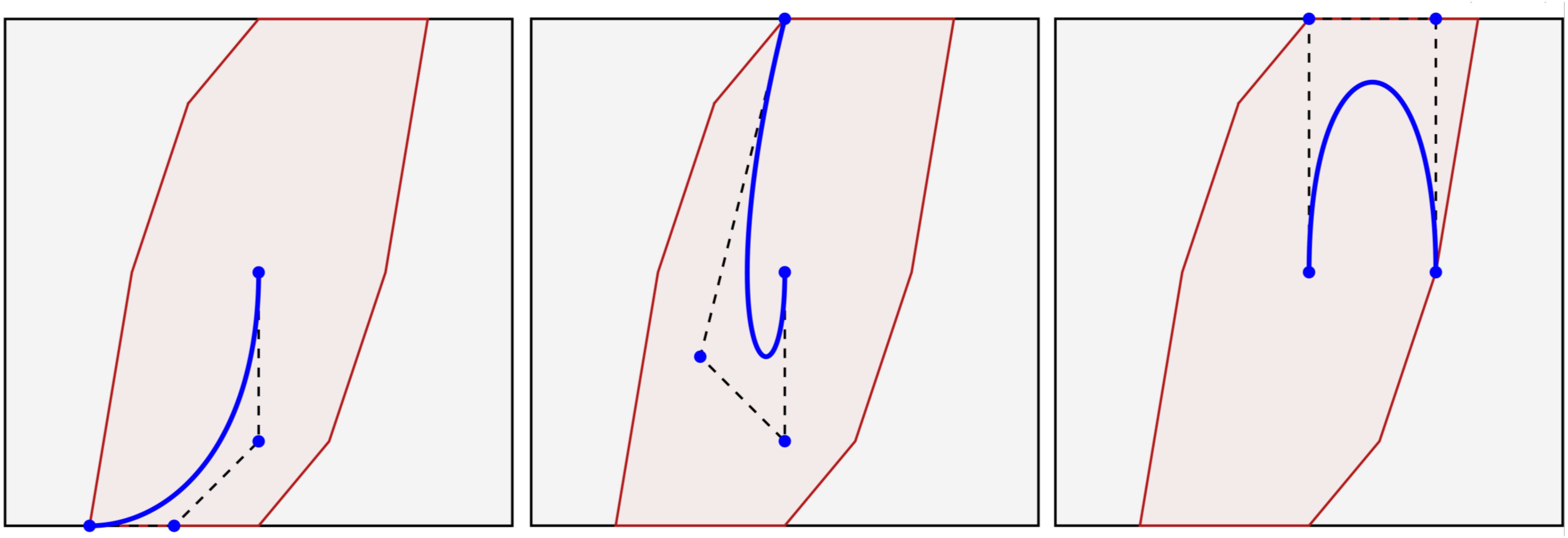
There exist matrices $\mathbf{F}$ and $\mathbf{G}$ such that any Bézier curve $\mathbf{B} : I \to \mathcal{X}_d$ with control points $\mathbf{p}$ satisfying:

$$\mathbf{F}\vec{\mathbf{p}} \leq \mathbf{G},$$

when tracked results in the closed loop system satisfying $\mathbf{x}(t) \in \mathcal{C}_{\mathcal{X}}$ and $\mathbf{k}(\mathbf{x}(t), \mathbf{x}_d, \mathbf{u}_d) \in \mathcal{C}_{\mathcal{U}}$ for all $t \in I$.

Bézier Reachable Polytopes

The set of Bézier curves satisfying $\mathbf{F}\vec{\mathbf{p}} \leq \mathbf{G}$ can be represented by a polytope.



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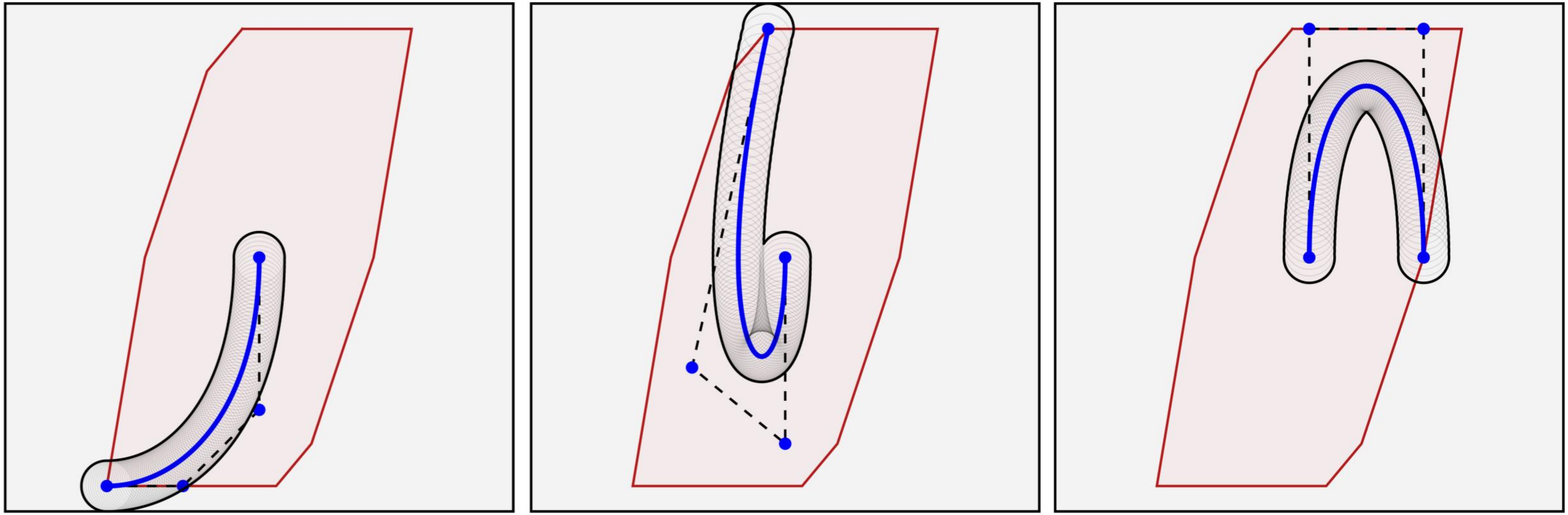

Bézier Reachable Polytopes

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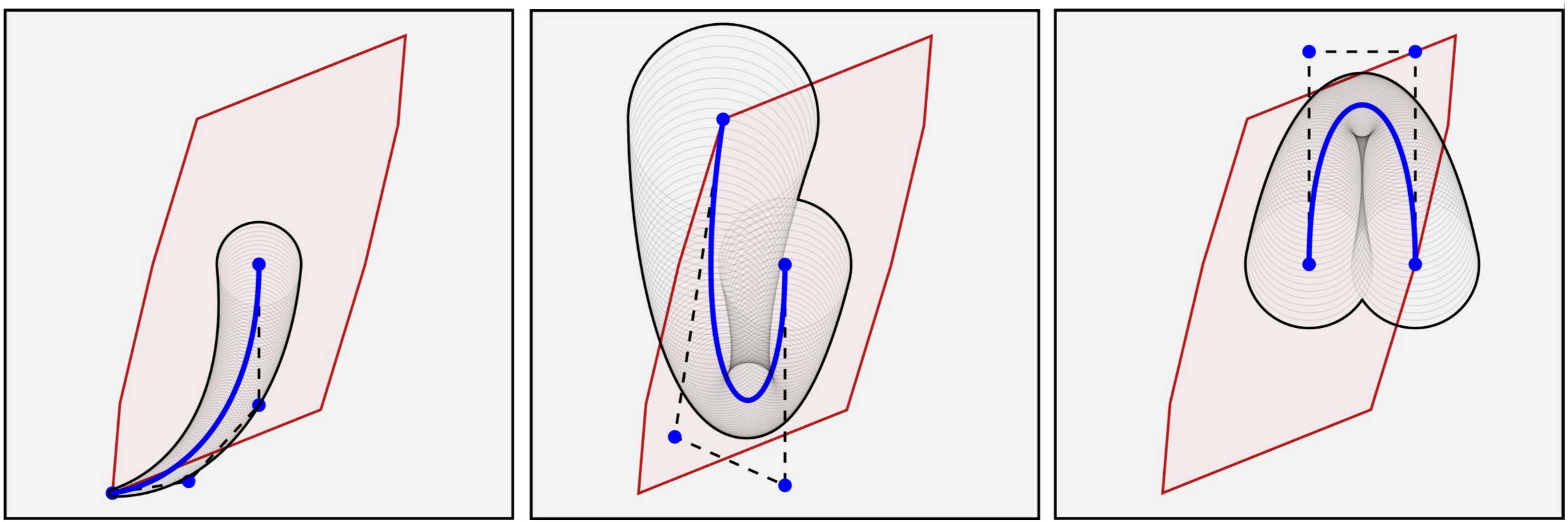
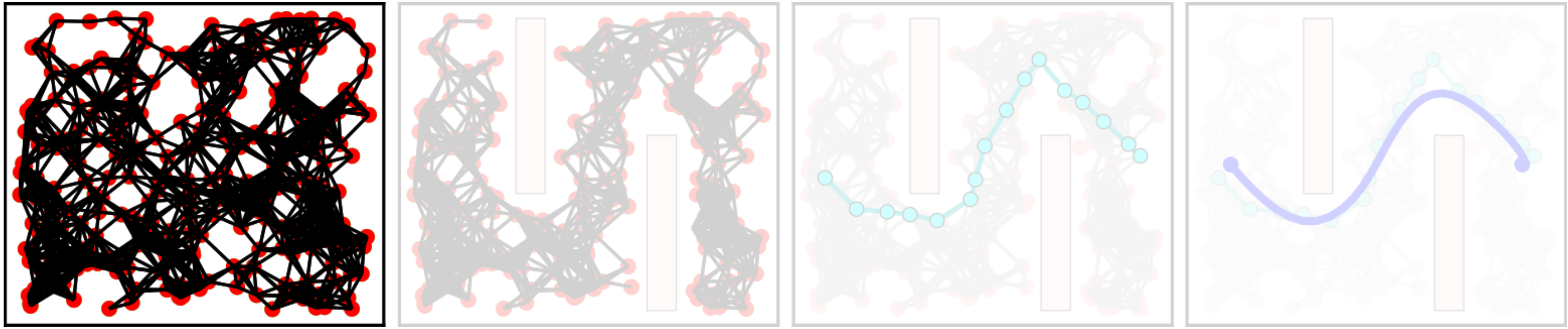


Path Planning



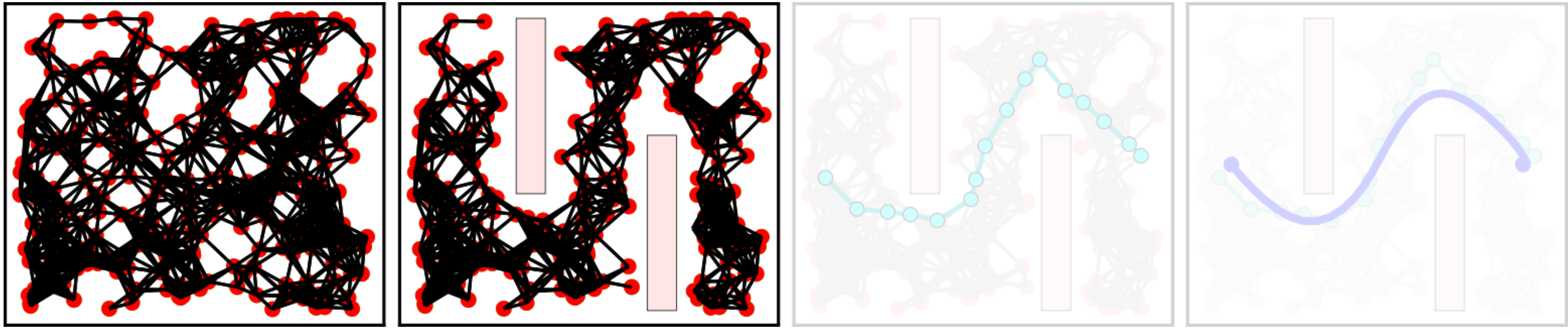
Build Bézier Graph

Cut Graph

Solve Graph

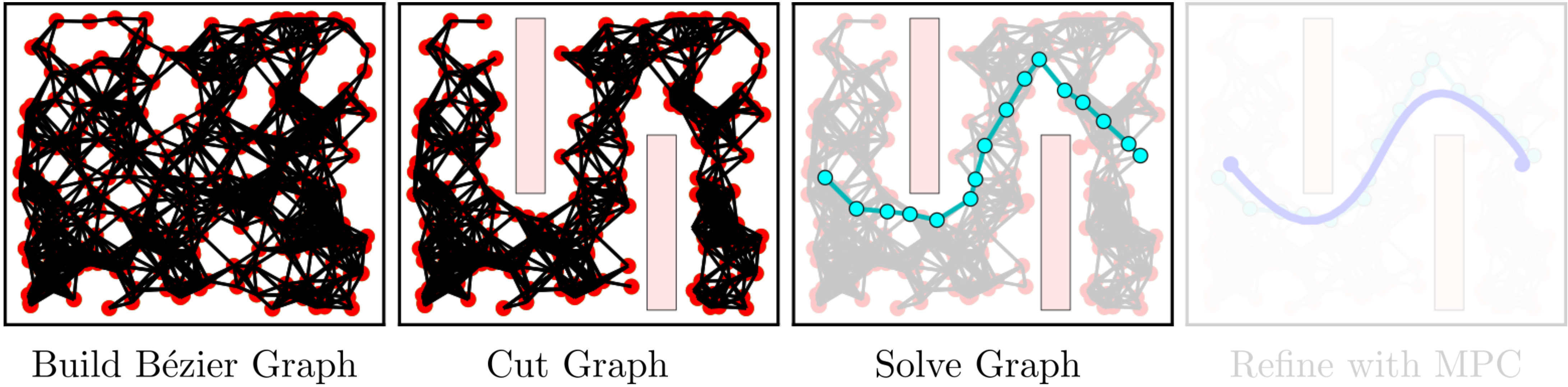
Refine with MPC

Path Planning

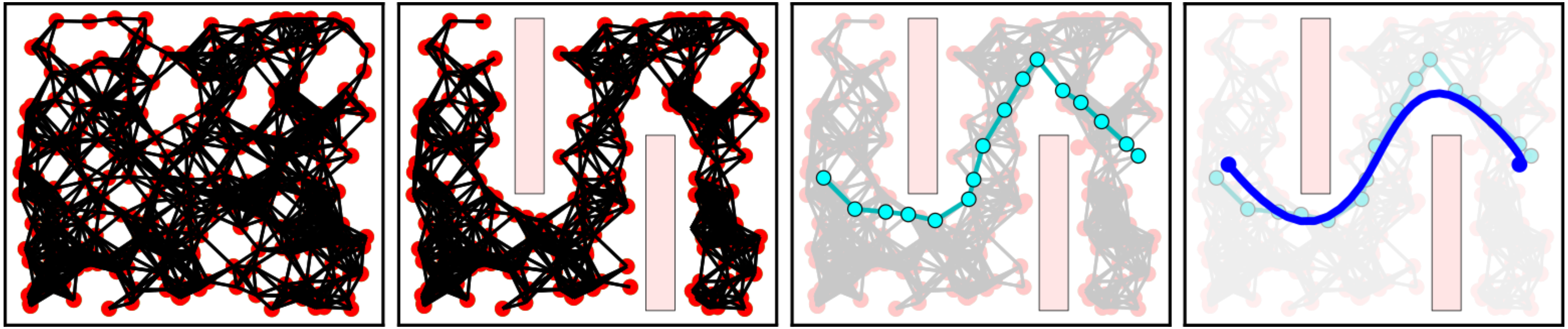


Build Bézier Graph | Cut Graph | Solve Graph | Refine with MPC

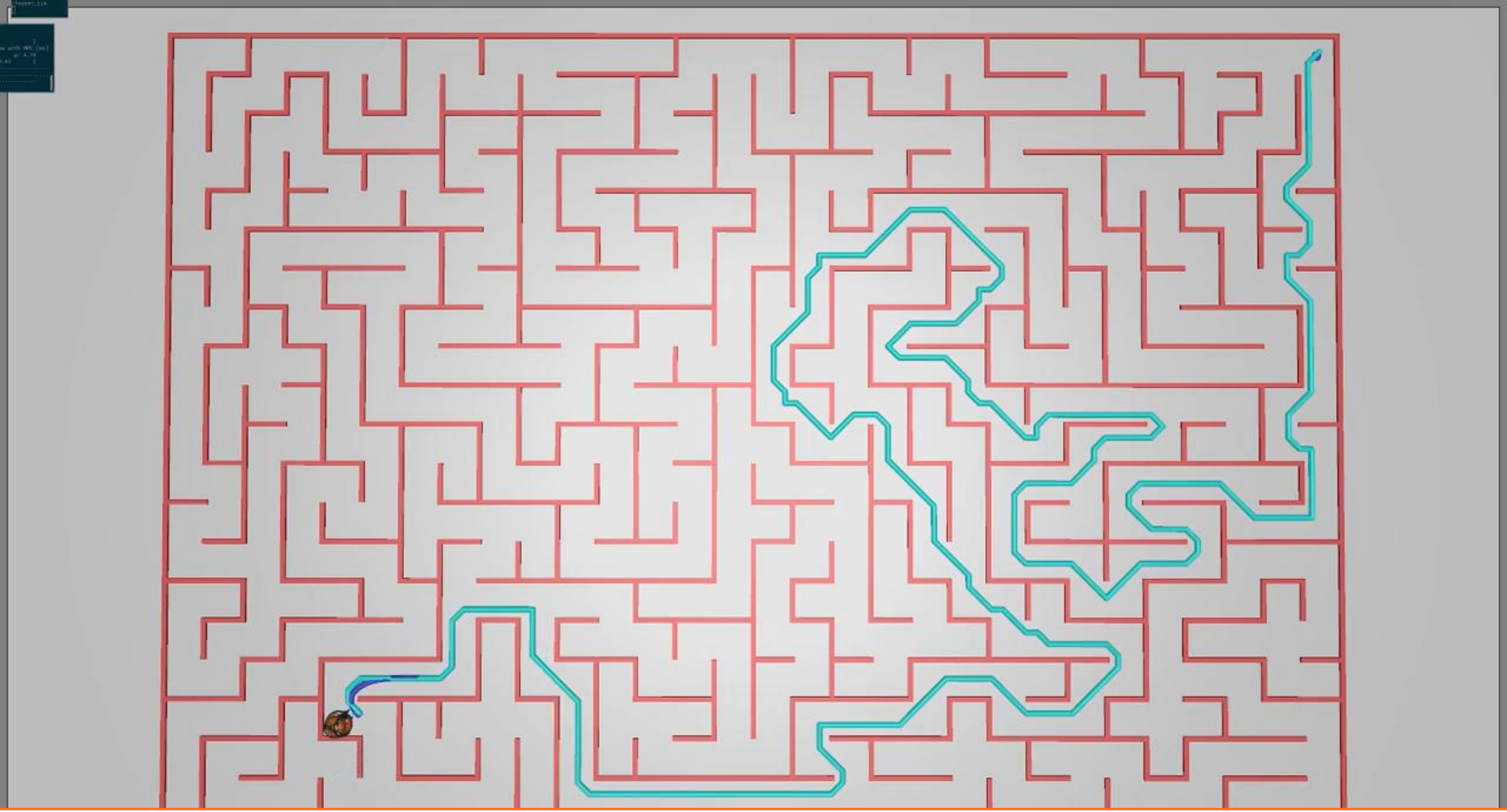
Path Planning



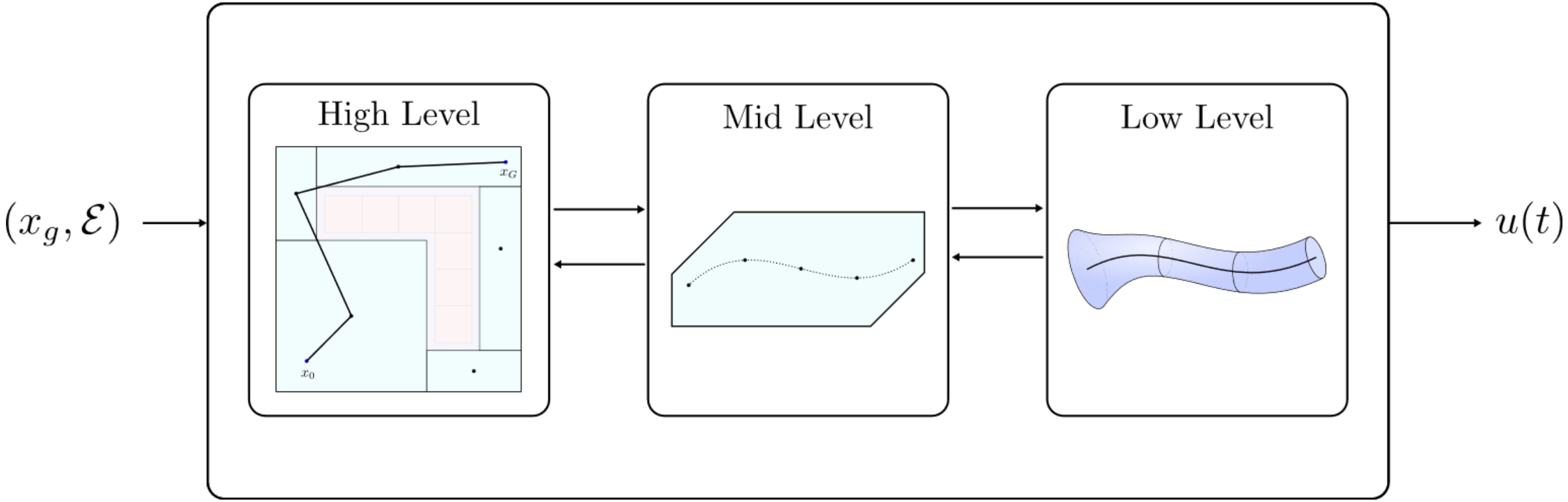
Path Planning



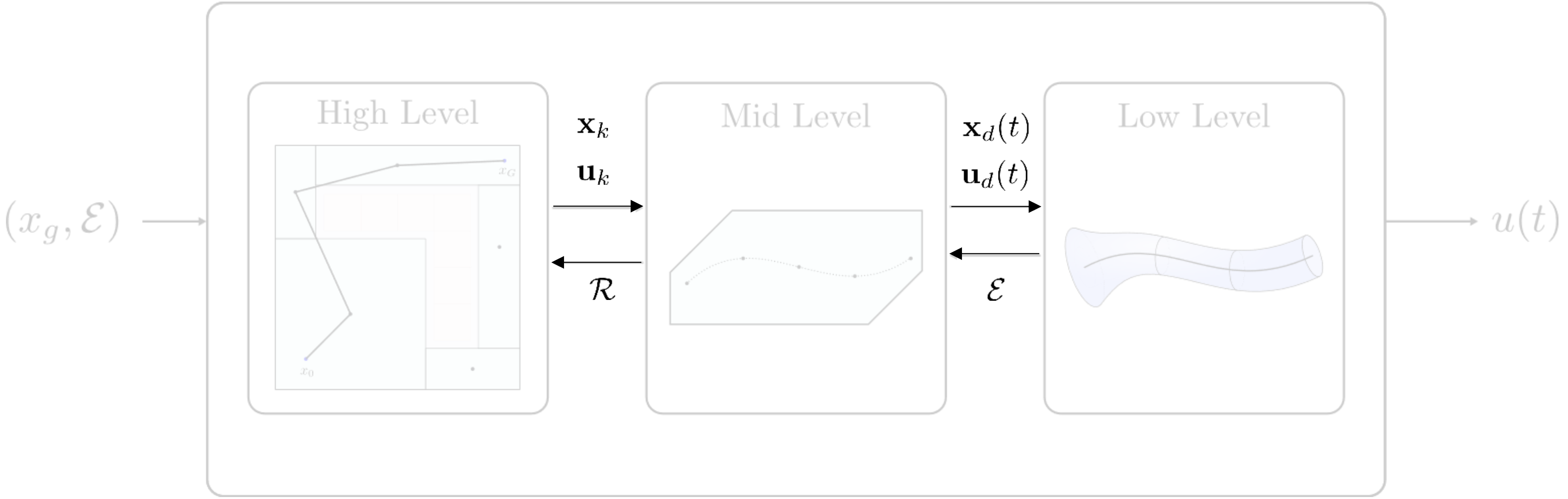
Build Bézier Graph Cut Graph Solve Graph Refine with MPC



A Theory of Hierarchies



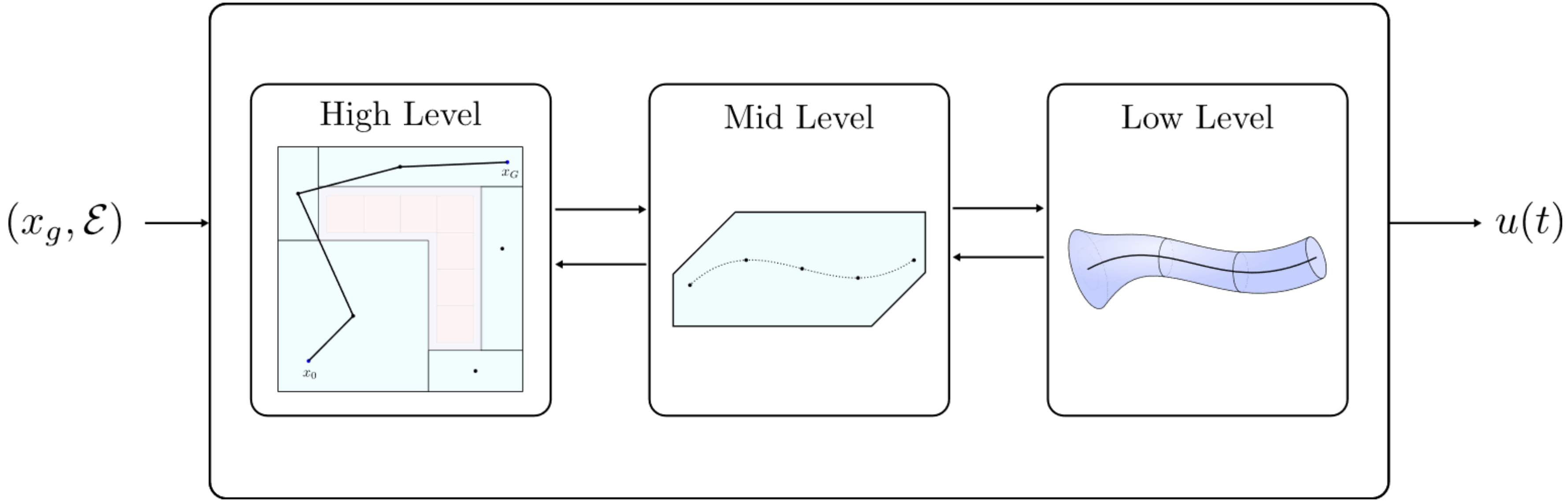
A Theory of Hierarchies



Setpoints

Certificates

Conclusion



Hierarchies are useful for:

- Efficiency
- Feasibility
- Generalizability

Thank You!

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