

# Fast Trajectory Optimization for Quadrupedal Walking on Slopes



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## Abstract

- Dimension reduction via the **CCS** framework leads to **faster gait generation**
- Experimentally feasible gait generation after **9.7 s** and **291 iterations**
- Bézier polynomial interpolation provides **stable gait transitions**
- Optimized gaits are **robust to variable outdoor terrain**

## Coupled Control Systems (CCSs)

$$\underbrace{D(q)\ddot{q} + H(q, \dot{q}) = u}_{\text{Full-Order Dynamics}} \Leftrightarrow \underbrace{\begin{cases} D_i(q_i)\ddot{q}_i + H_i(q_i, \dot{q}_i) = u_i + J_{h_i}^\top \lambda \\ \text{s.t. } h_i(q) = 0 \end{cases}}_{\text{Reduced-Order Coupled Dynamics}}$$

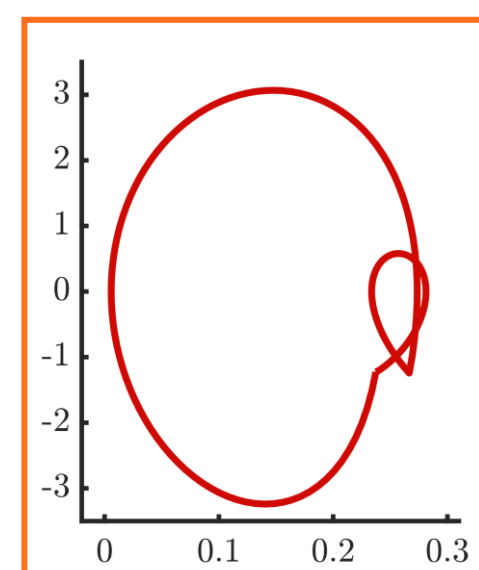
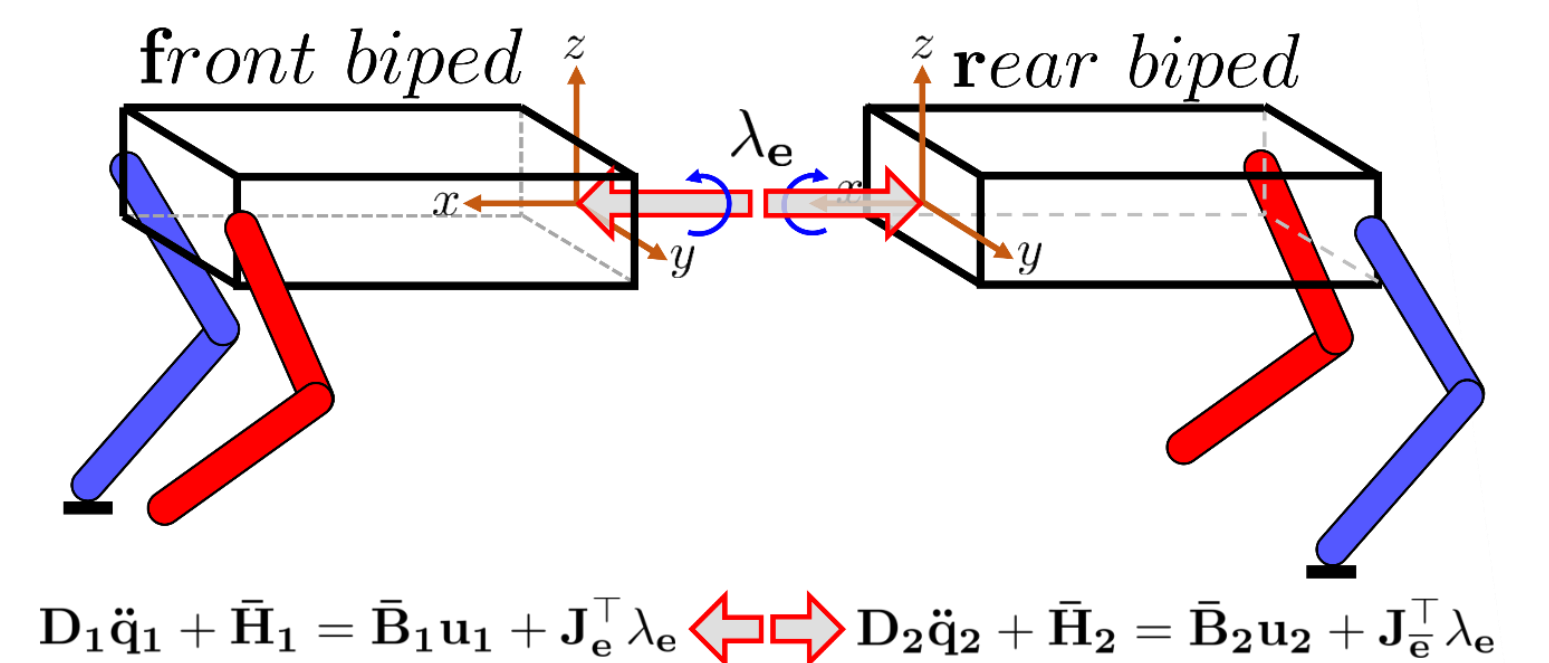
## Gait Generation

$$\min_{\{\vartheta^\kappa\}_{\kappa=0, \dots, K}} \sum_{\kappa} \left\| \dot{\xi}_f^\kappa \right\|_2^2 \quad \kappa = 0, 1 \dots K$$

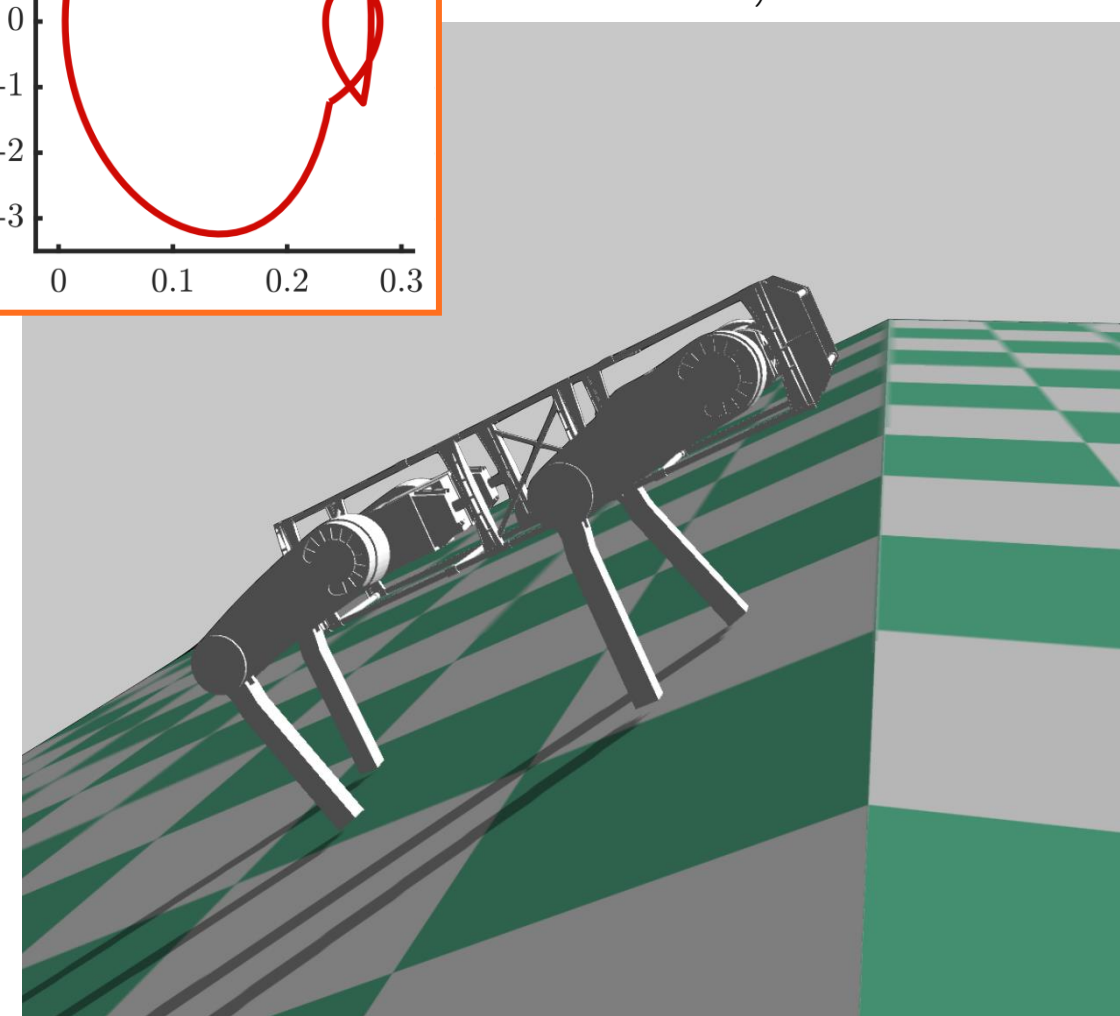
- s.t.
- (C.1) dynamic collocation constraints
  - (C.2) periodic constraints
  - (C.3) path constraints
  - (C.4) feasibility constraints

## Gait Transitions

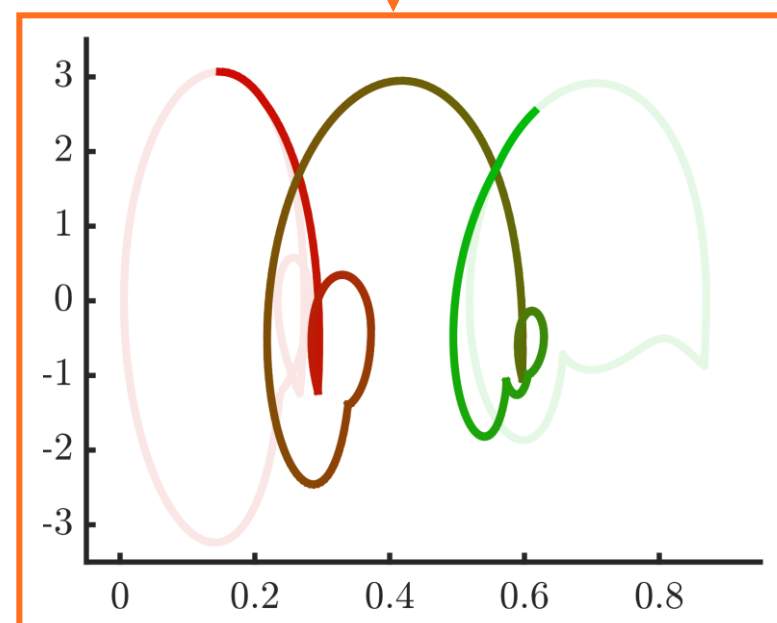
$$\begin{aligned} q_i^d(t) &= \mathcal{B}_i(t) \\ q^d &= (1 - \eta)q_{i-1}^d + \eta q_i^d \\ u &= -k_p(q^a - q^d) - k_d(\dot{q}^a - \dot{q}^d) \end{aligned}$$



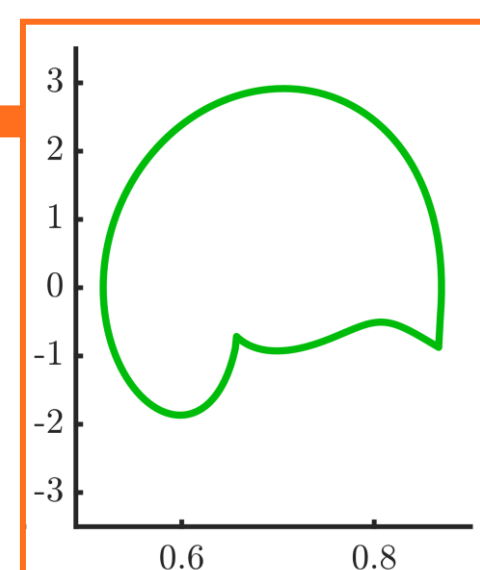
9.8 s, 264 iter



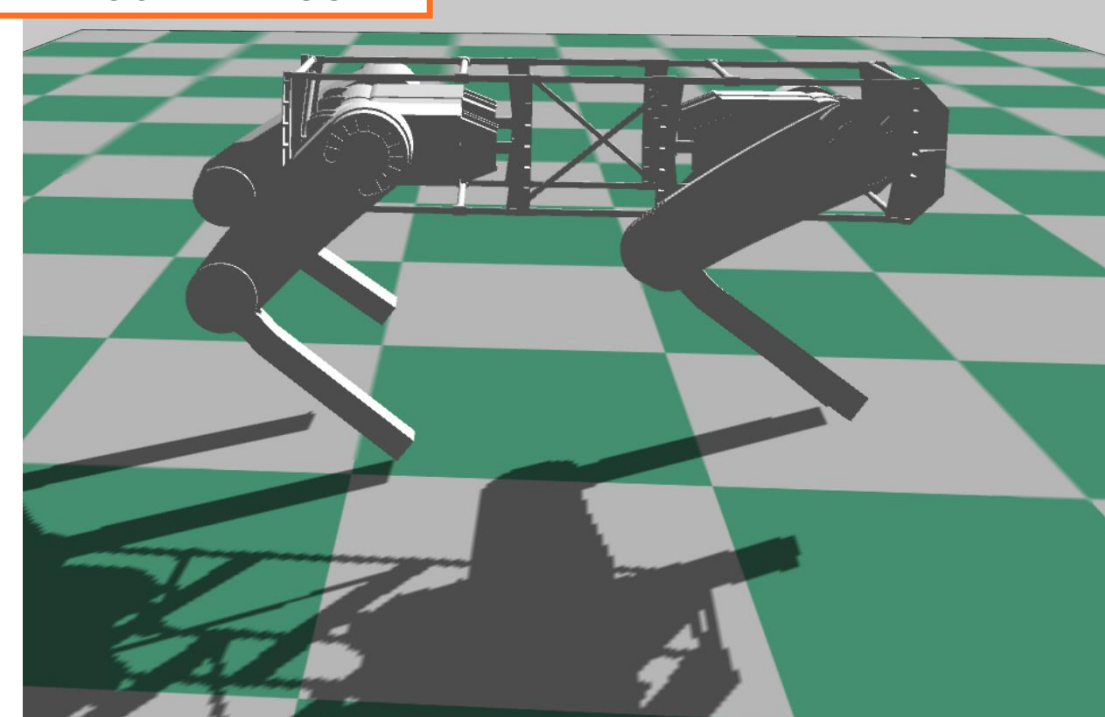
30° Uphill



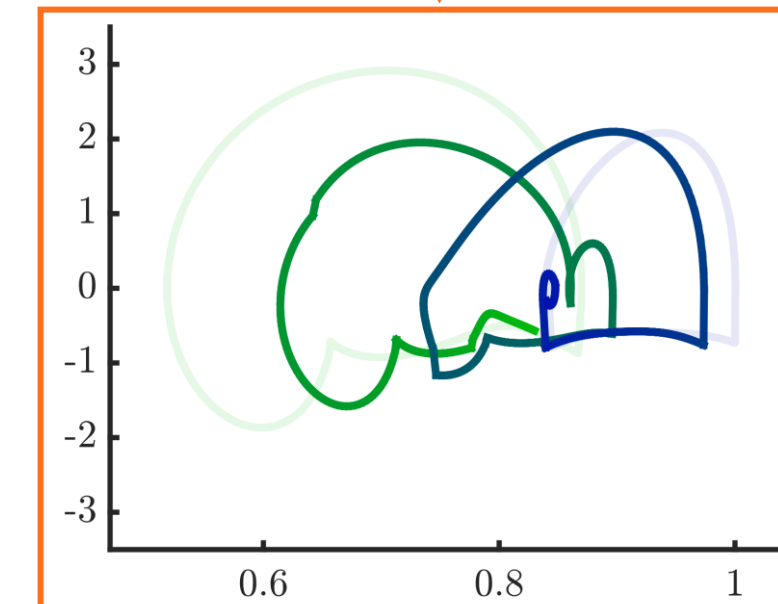
Transition 1



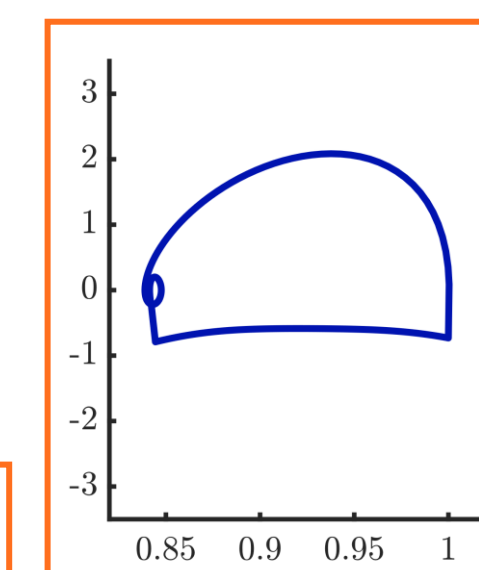
17.2 s, 295 iter



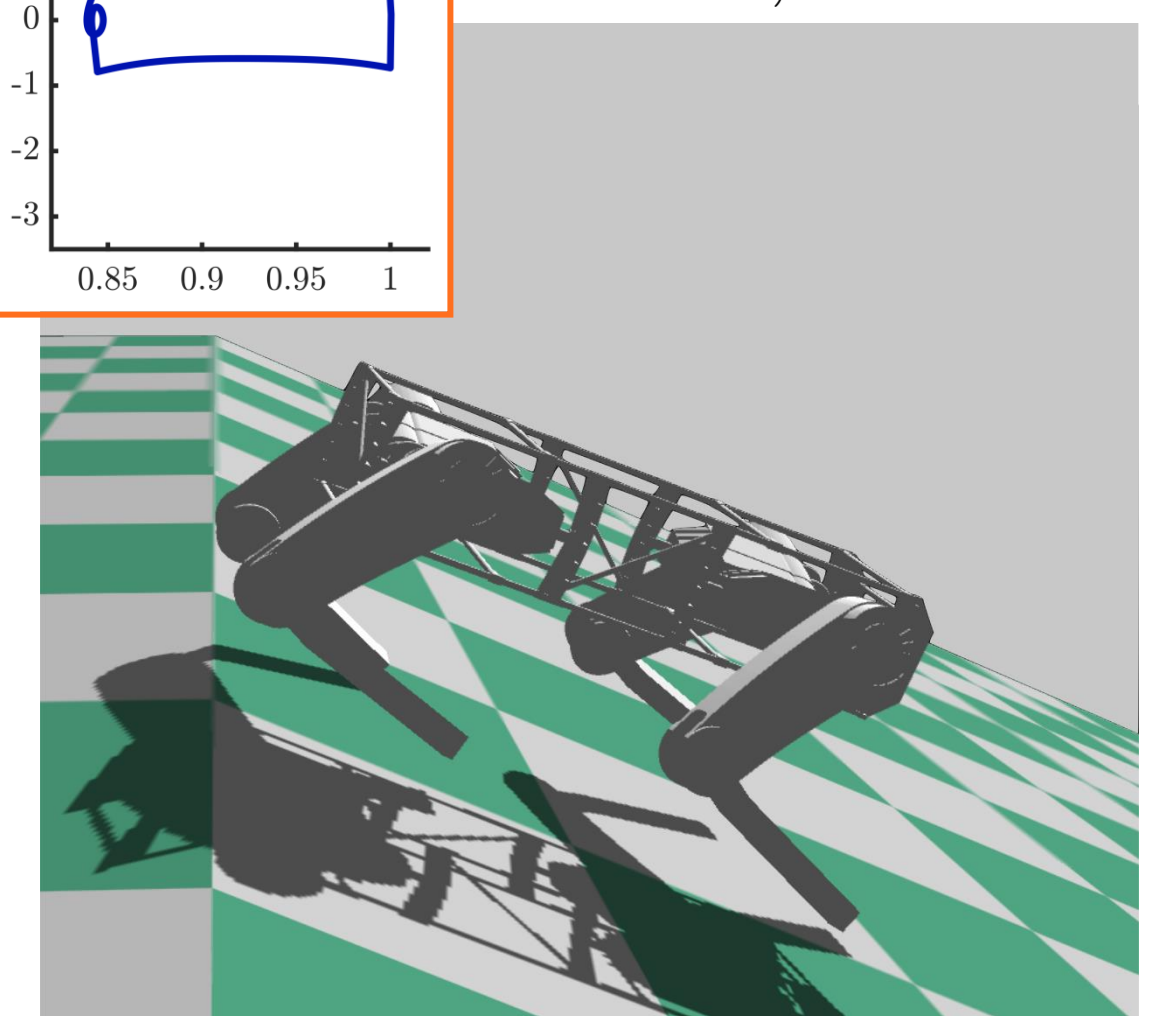
Level Ground



Transition 2

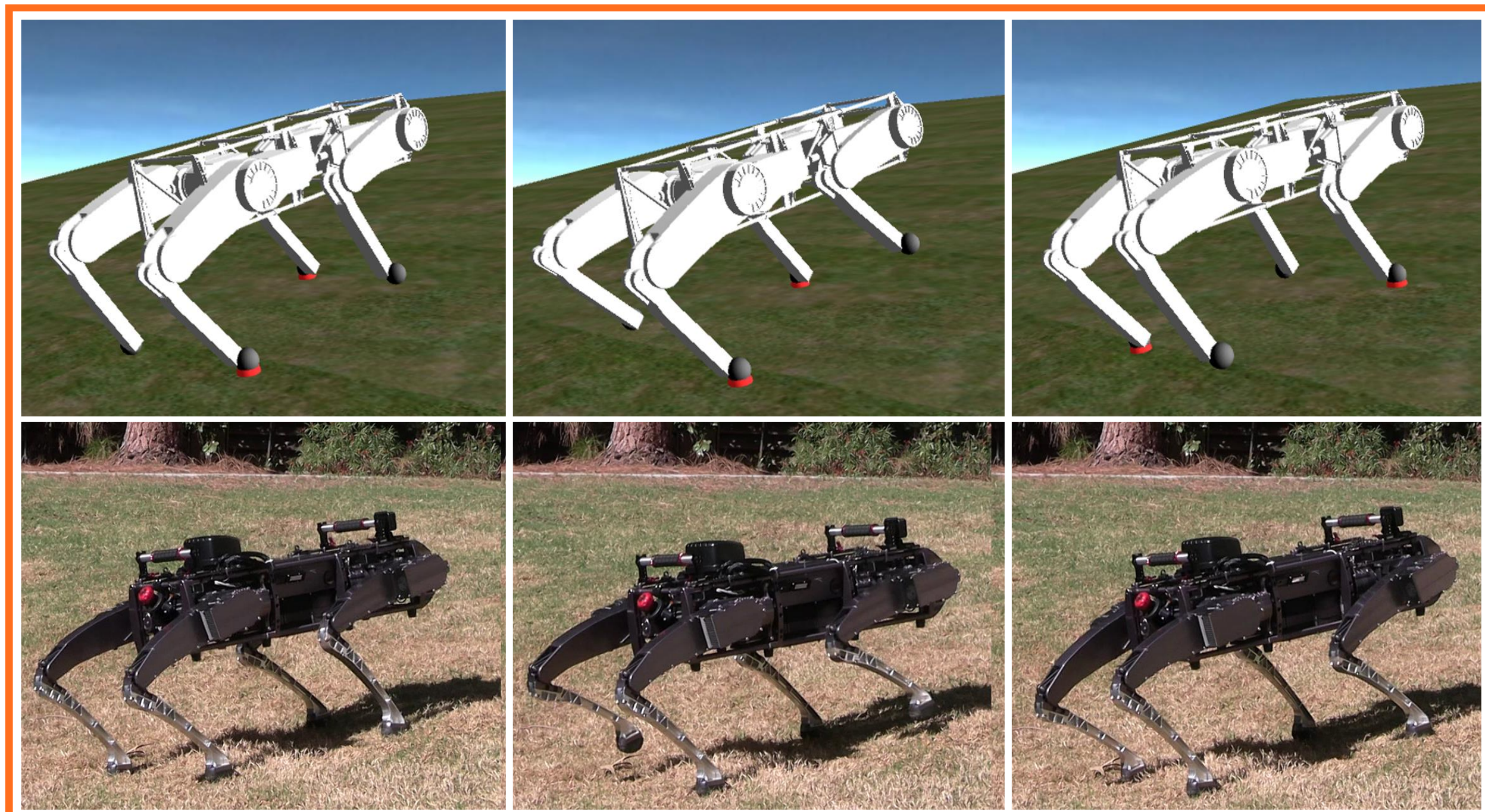


4.2 s, 125 iter



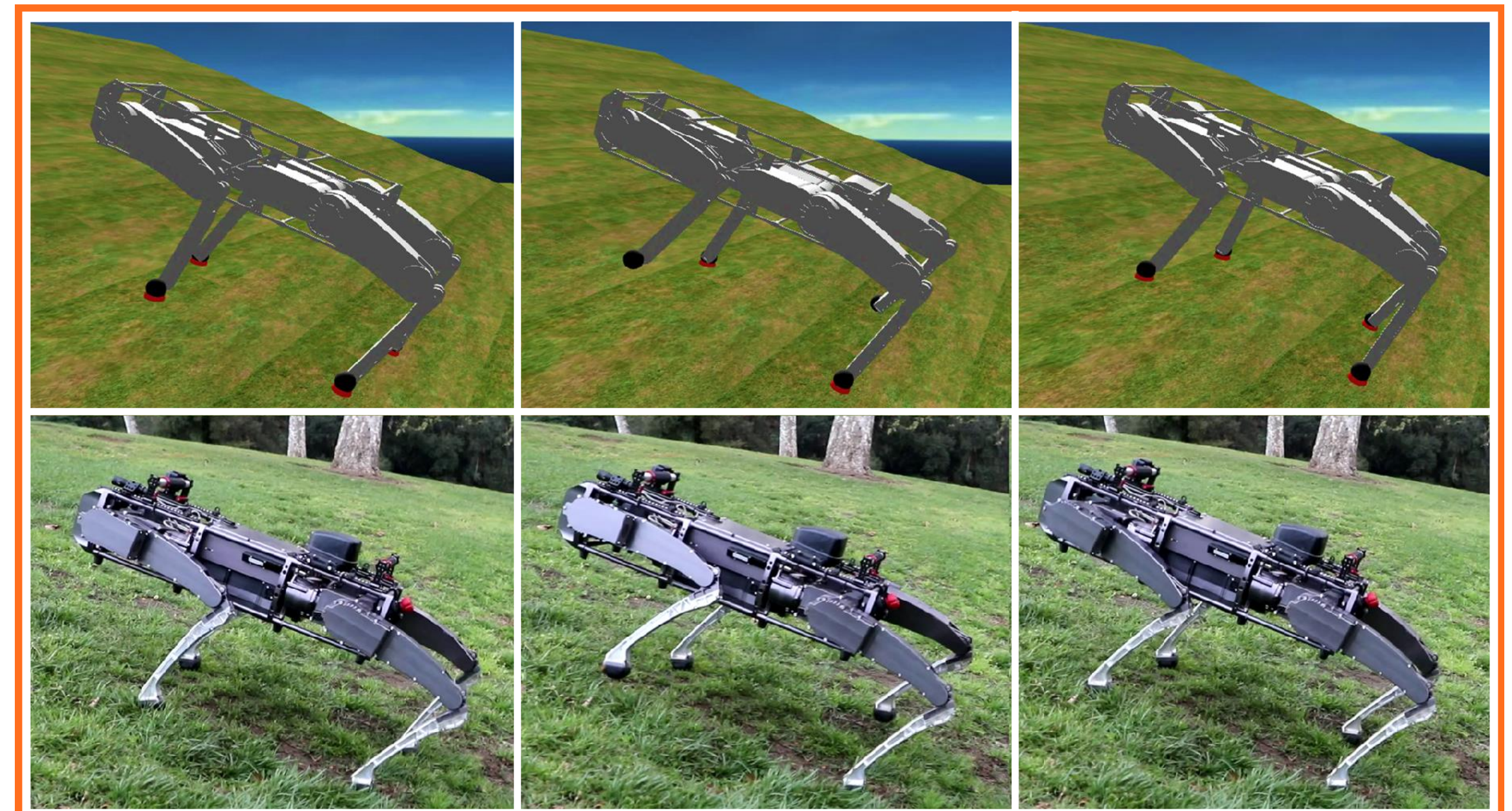
20° Downhill

## Outdoor Testing



Walking Outside: <https://youtu.be/Cp9XbWkS24U>

## Simulation



Simulation: [https://youtu.be/\\_xrW1Mc7e0c](https://youtu.be/_xrW1Mc7e0c)